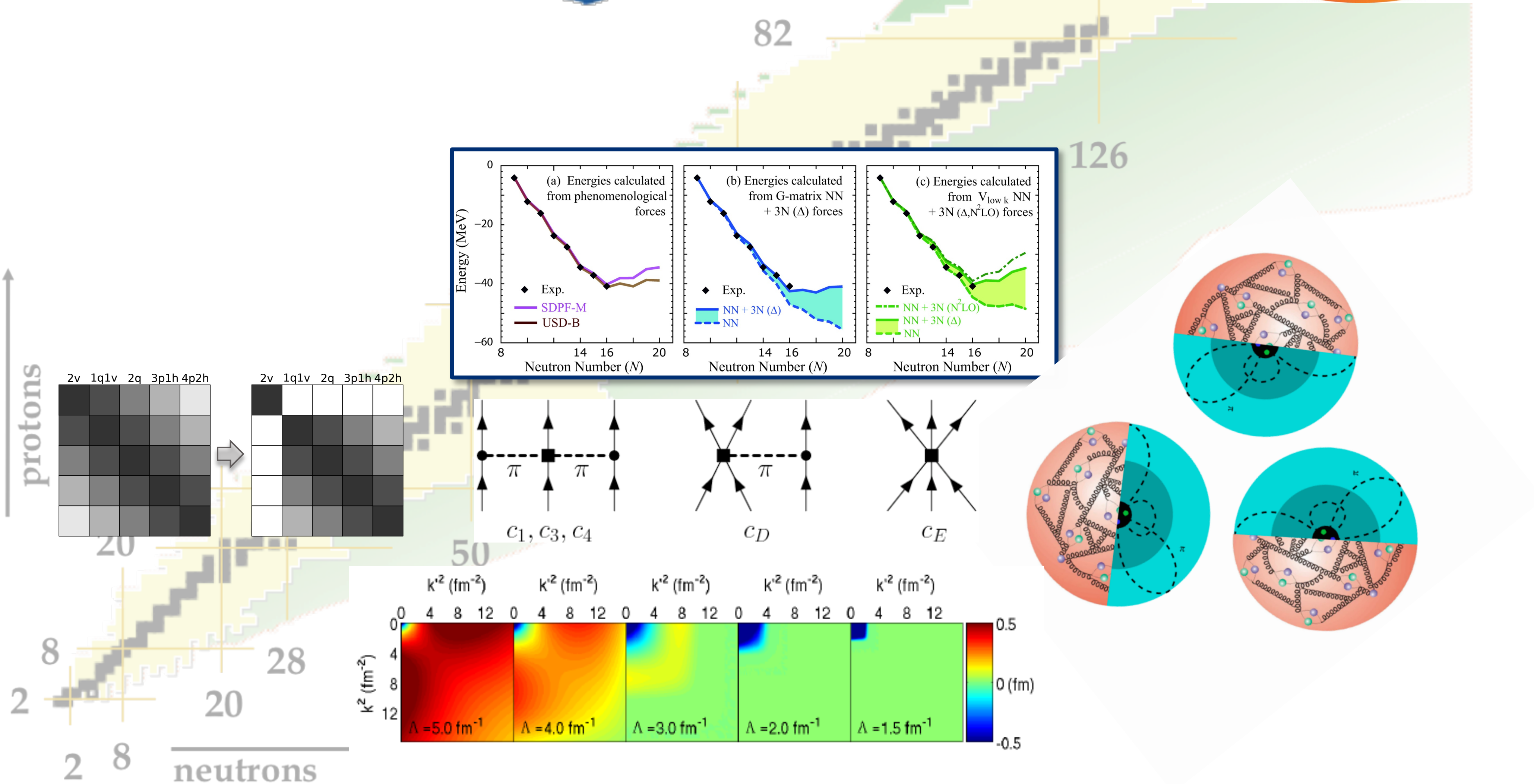
# Medium-Mass Nuclei from First Principles


Jason D. Holt

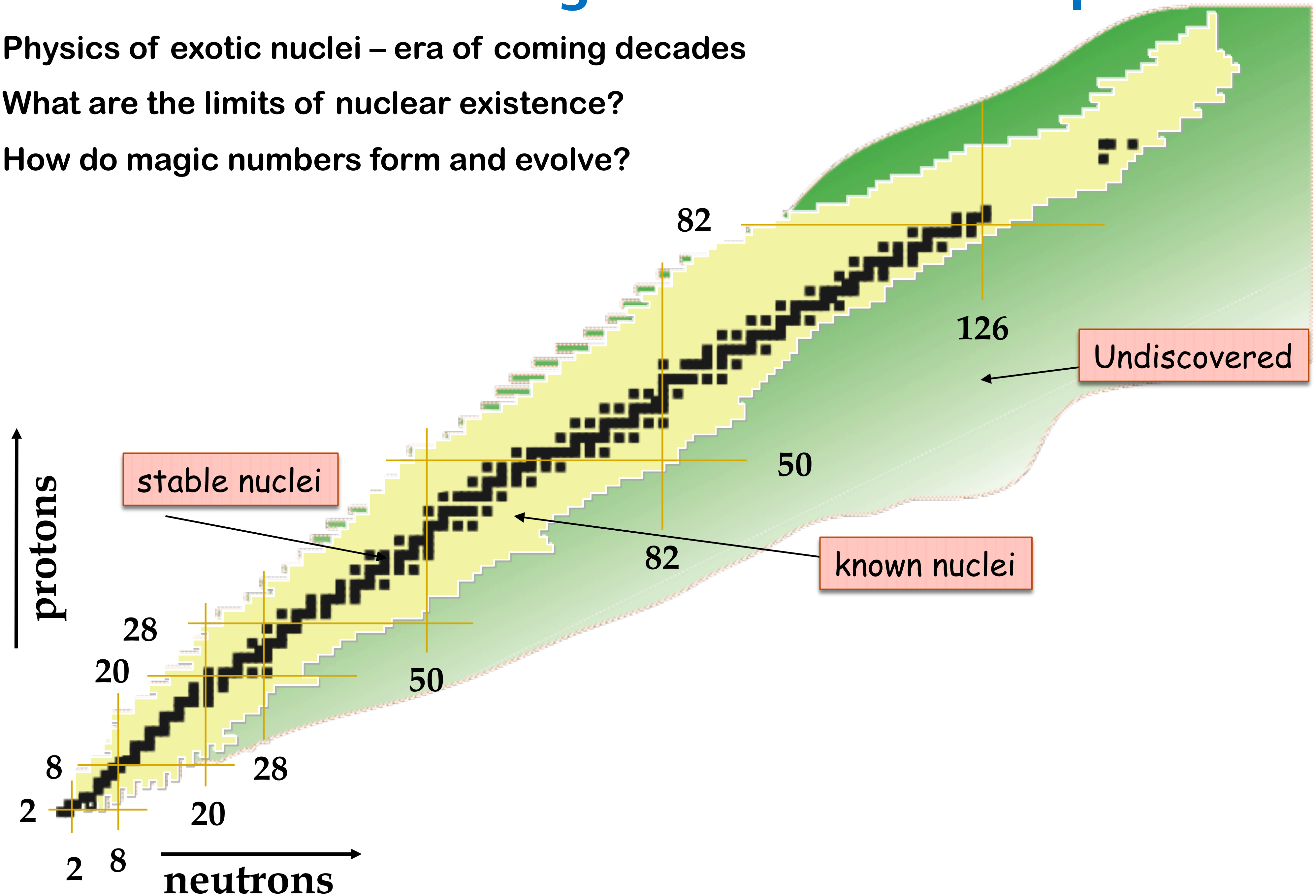



# Drip Lines and Magic Numbers: The Evolving Nuclear Landscape

Physics of exotic nuclei – era of coming decades

What are the limits of nuclear existence?

How do magic numbers form and evolve?



# Drip Lines and Magic Numbers: 3N Forces in Medium-Mass Nuclei

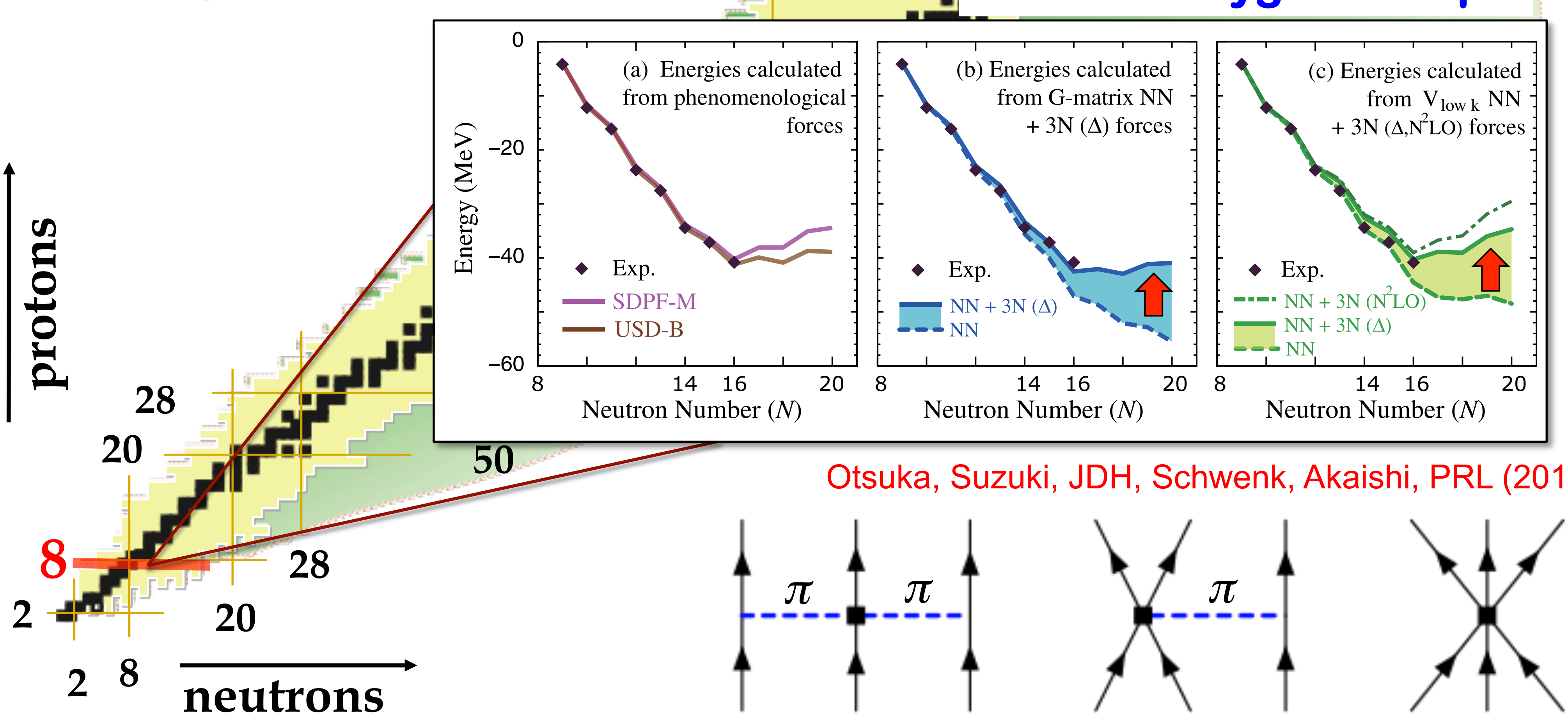
**Exploring the frontiers of nuclear science:**

Worldwide joint experimental/theoretical effort

What are the properties of proton/neutron-rich matter?

**What are the limits of nuclear existence?**

How do magic numbers form and evolve?




Otsuka, Suzuki, JDH, Schwenk, Akaishi, PRL (2010)

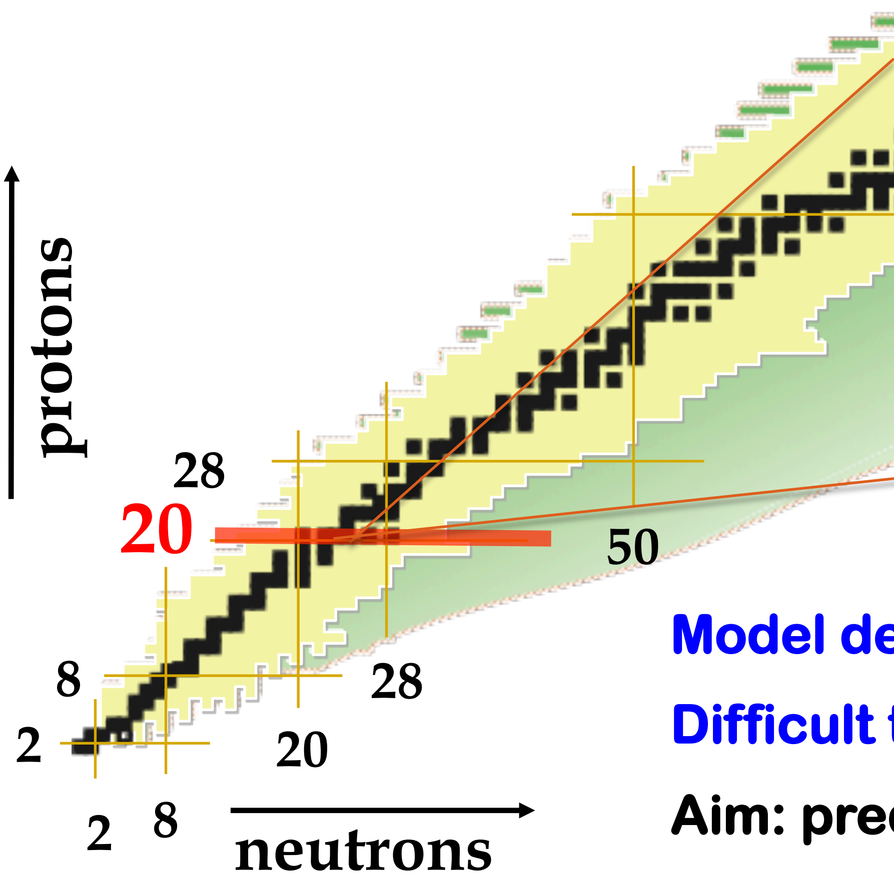
# Limits of Empirical Approach

What are the properties of proton/neutron-rich matter?

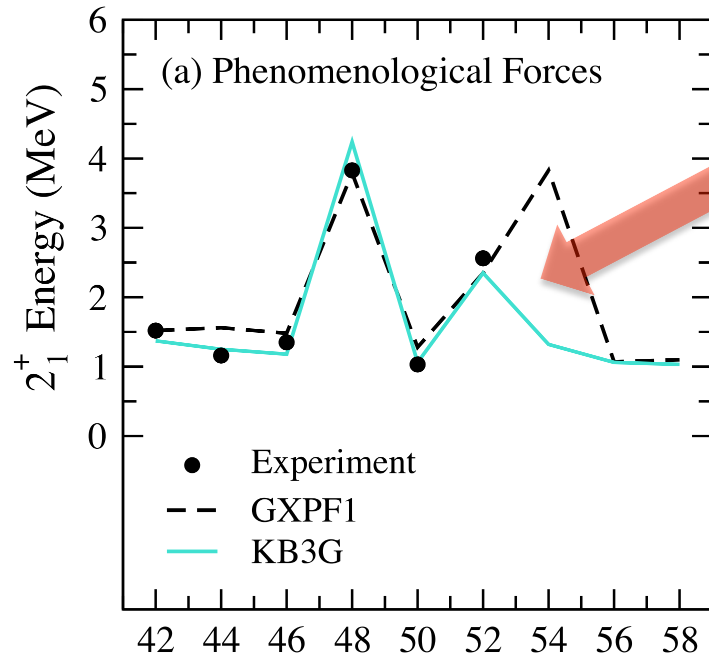
What are the limits of existence of matter?

How do magic numbers form and evolve?

Worldwide joint experimental/theoretical effort!




**N=34 magic number in calcium?**




**Model dependent extrapolations**

**Difficult to extract physics: continuum, deformation**

**Aim: predictive ab initio theory far from stability**

# Drip Lines and Magic Numbers:
# 3N Forces in Medium-Mass Nuclei

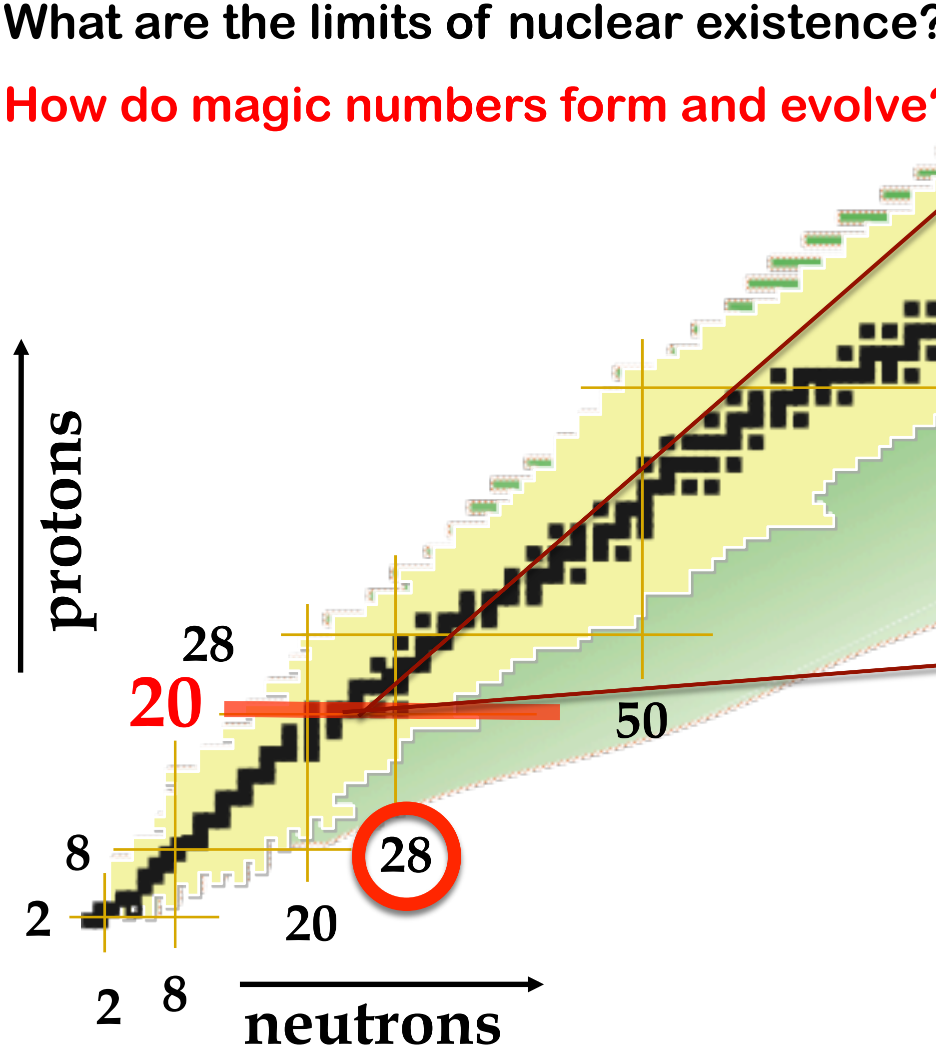
**Exploring the frontiers of nuclear science:**

Worldwide joint experimental/theoretical effort

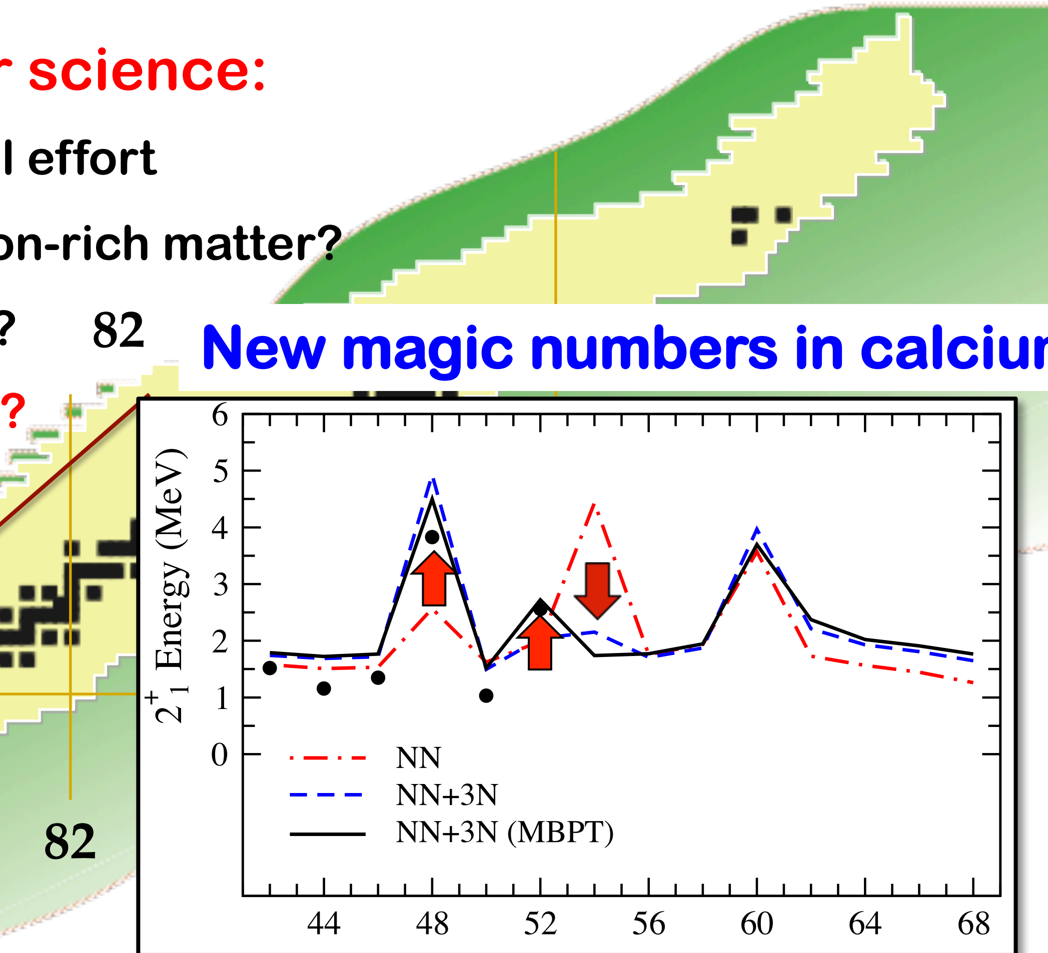
What are the properties of proton/neutron-rich matter?

What are the limits of nuclear existence?

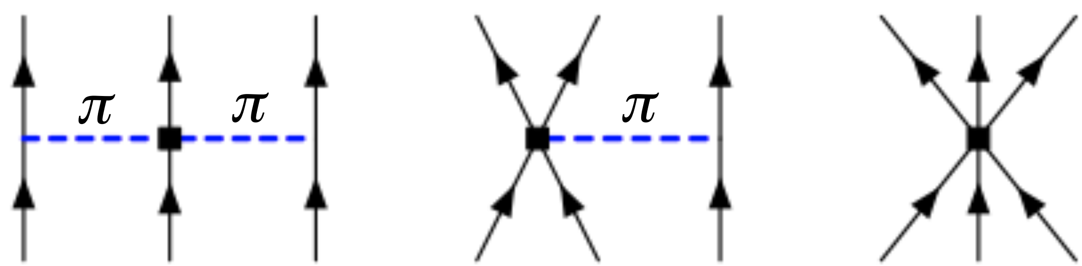
**How do magic numbers form and evolve?**




**New magic numbers in calcium**




JDH, Otsuka, Schwenk, Suzuki, JPG (2012)
JDH, Menendez, Schwenk, JPG (2013)



# Drip Lines and Magic Numbers: 3N Forces in Medium-Mass Nuclei

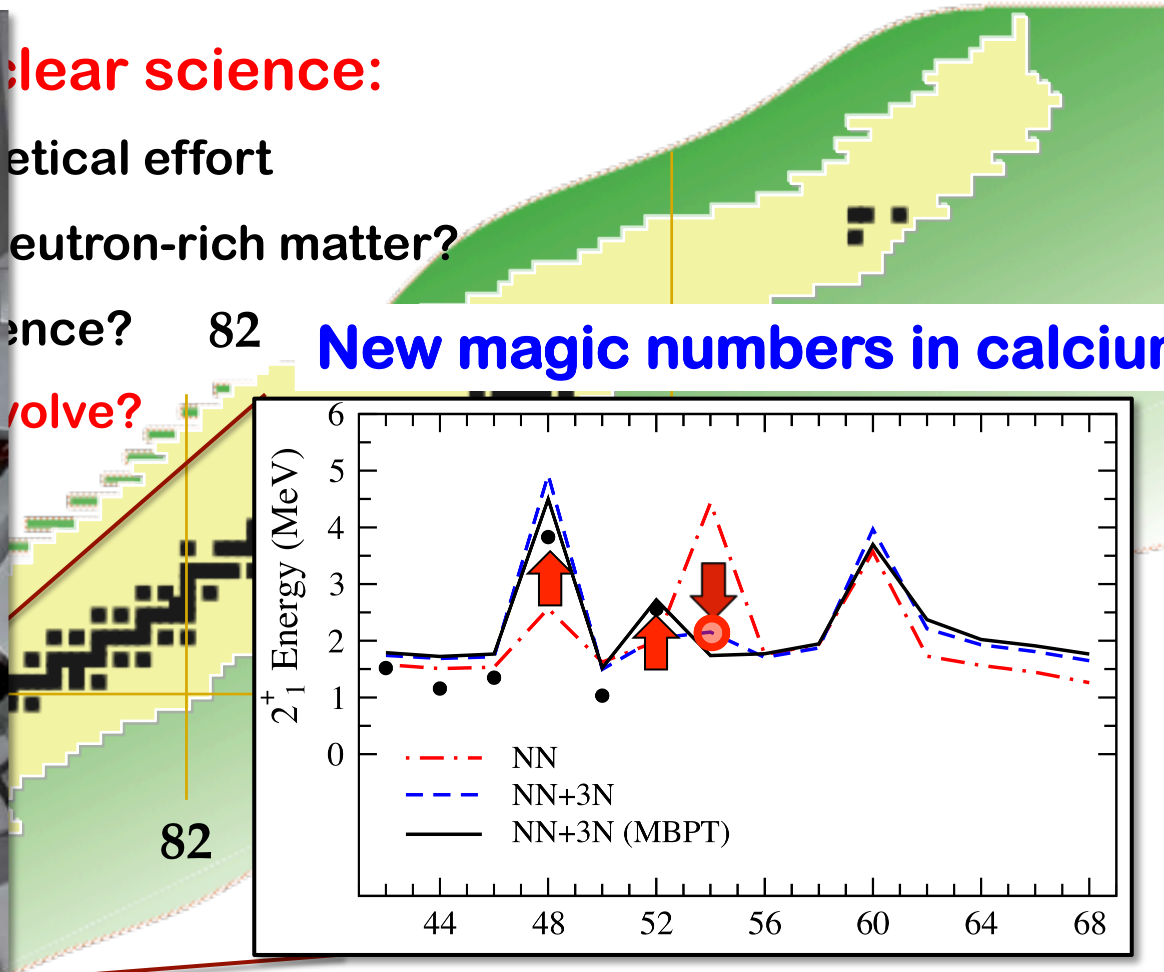

clear science:

etical effort

eutron-rich matter?

ence?

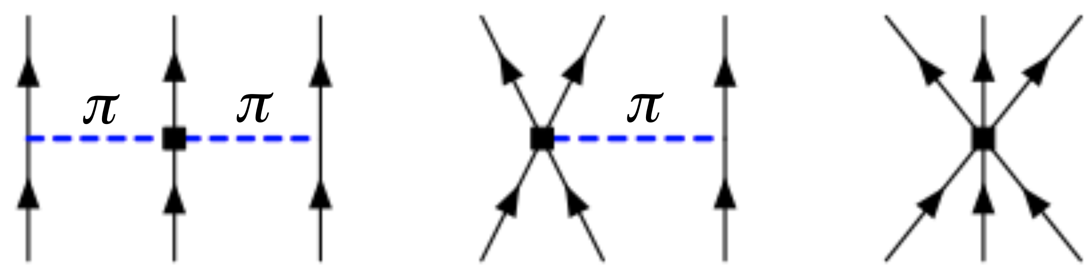
volve?



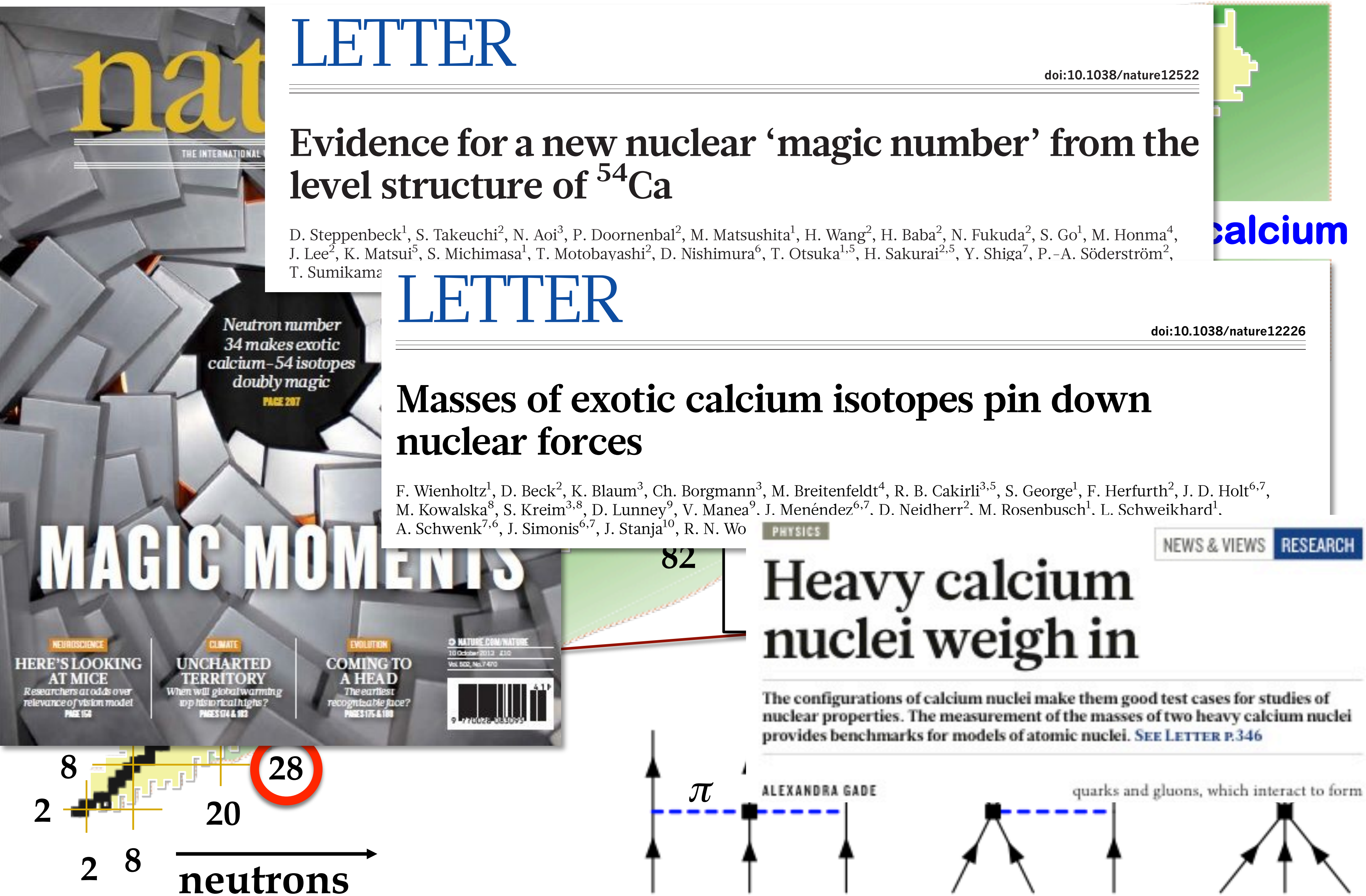

**New magic numbers in calcium**



JDH, Otsuka, Schwenk, Suzuki, JPG (2012)
JDH, Menendez, Schwenk, JPG (2013)



# Drip Lines and Magic Numbers:
# 3N Forces in Medium-Mass Nuclei



# Approaches to Nuclear Structure

"The first, the basic approach, is to study the elementary particles, their properties and mutual interaction. Thus one hopes to obtain knowledge of the nuclear forces. If the forces are known, one should, in principle, be able to calculate deductively the properties of individual nuclei. Only after this has been accomplished can one say that one completely understands nuclear structure…

The other approach is that of the experimentalist and consists in obtaining by direct experimentation as many data as possible for individual nuclei. One hopes in this way to find regularities and correlations which give a clue to the structure of the nucleus... The shell model, although proposed by theoreticians, really corresponds to the experimentalist's approach."

*–M. Goeppert-Mayer, Nobel Lecture*

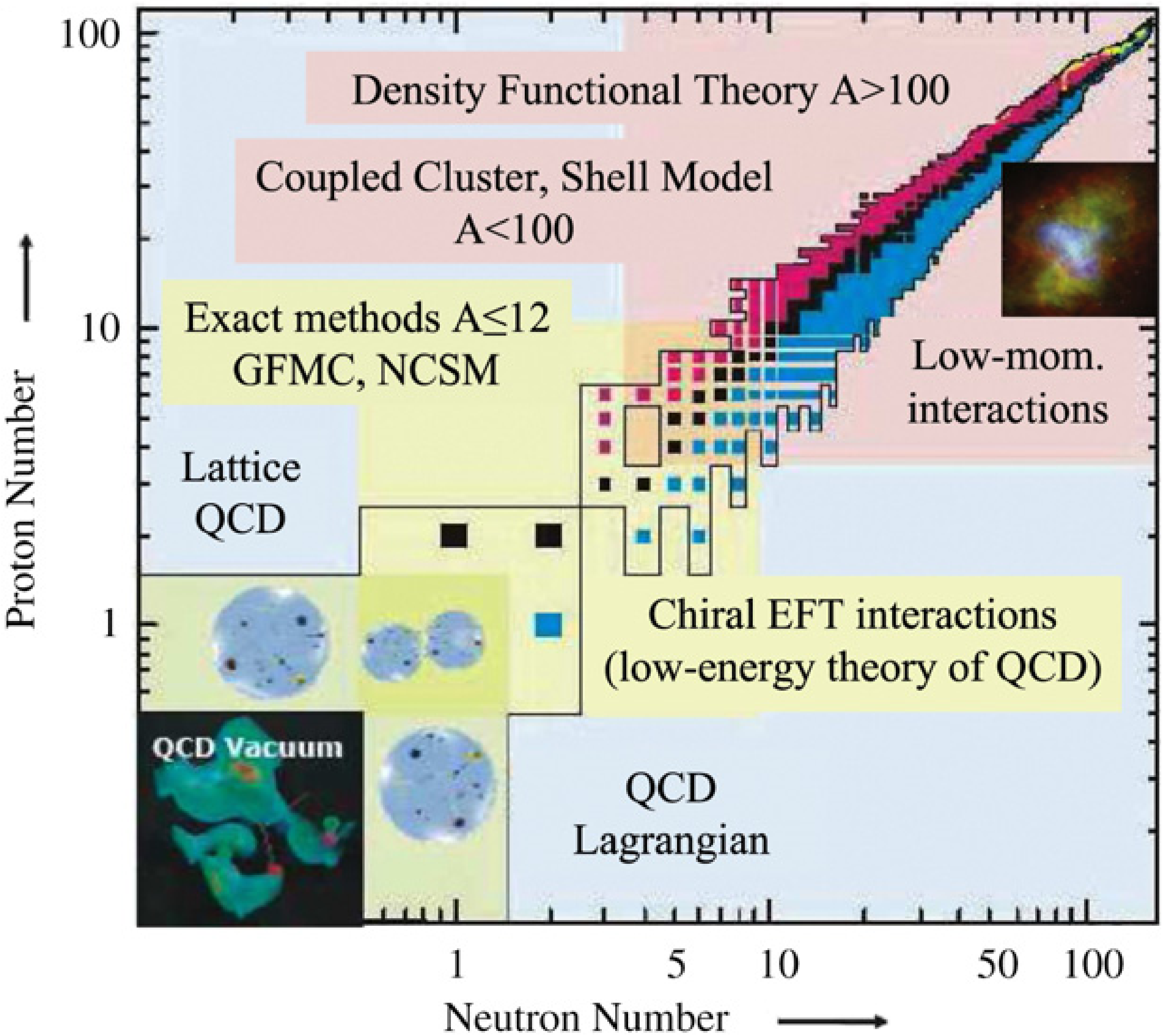

*Ab initio* approach vs. *phenomenological*

Theories of medium-mass nuclei largely empirical

*Purpose of lectures is to show how shell model can be based on the first approach!*

# The Challenge of Ab Initio Nuclear Theory

To understand the properties of complex nuclei from first principles




Two significant issues:

**Interaction**

Not well understood

Not obtainable from QCD

Too "hard" to be useful

Multiple energy scales

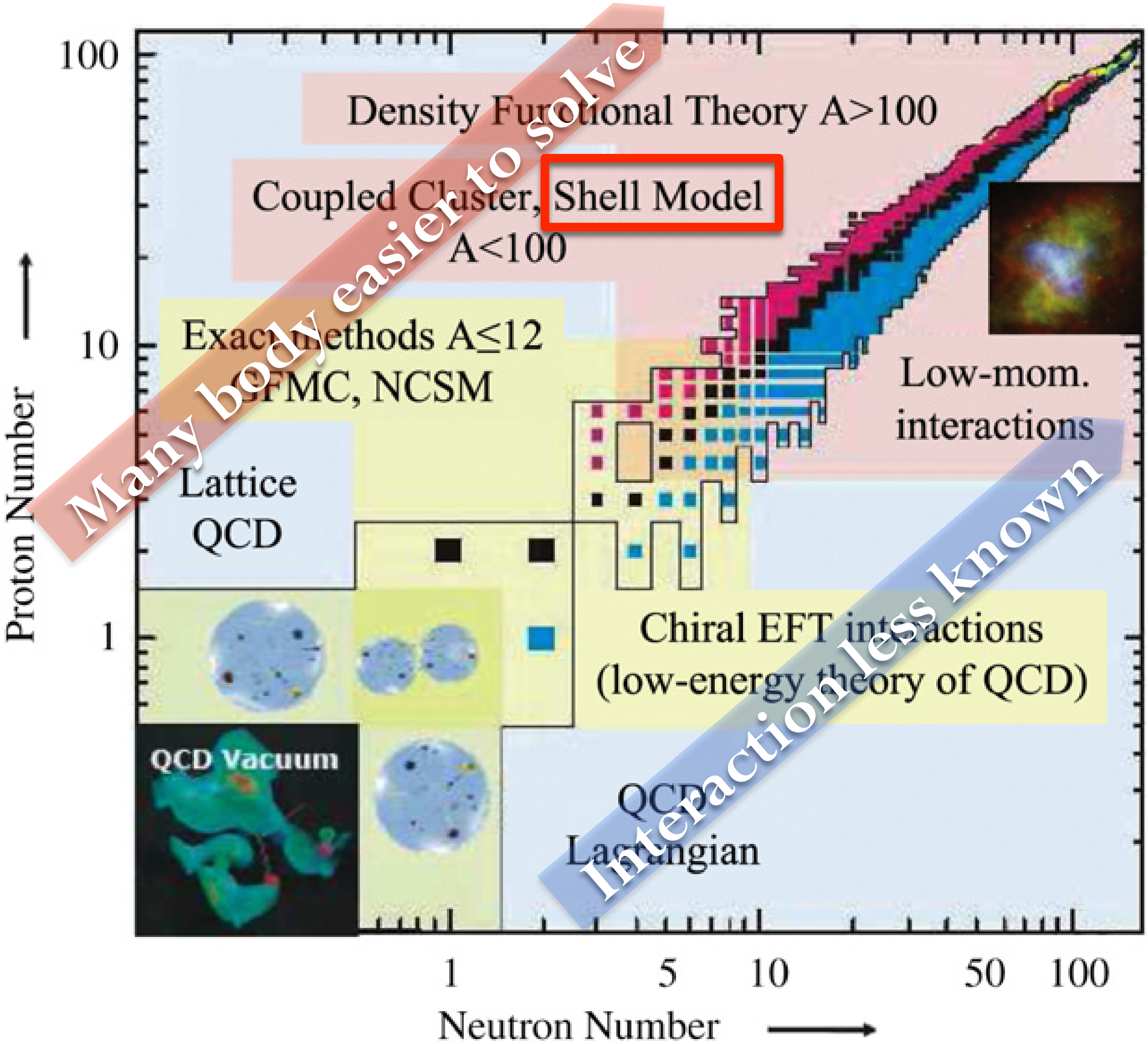
**Many-body Problem**

Not 'exactly' solvable above $A \sim 20$

Here we focus on shell model

# The Challenge of Ab Initio Nuclear Theory

To understand the properties of complex nuclei from first principles




Two significant issues:

**Interaction**

Not well understood

Not obtainable from QCD

Too "hard" to be useful

Multiple energy scales

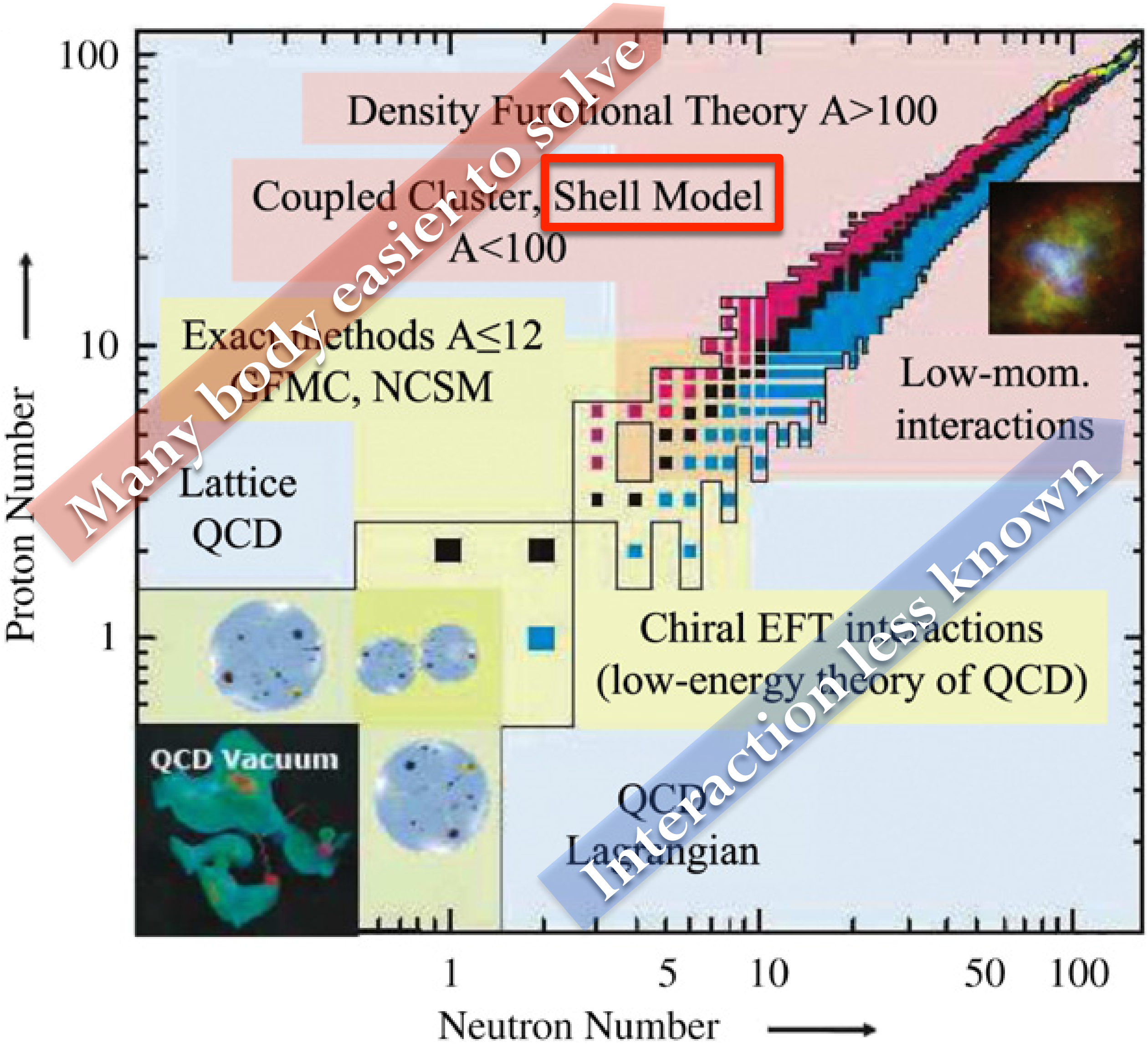
**Many-body Problem**

Not 'exactly' solvable above $A \sim 20$

Here we focus on shell model

# The Challenge of Ab Initio Nuclear Theory

To understand the properties of complex nuclei from first principles




Two significant issues:

**Interaction**

Not well understood

Not obtainable from QCD

Too "hard" to be useful

Multiple energy scales

**Many-body Problem**

Not 'exactly' solvable above $A \sim 20$

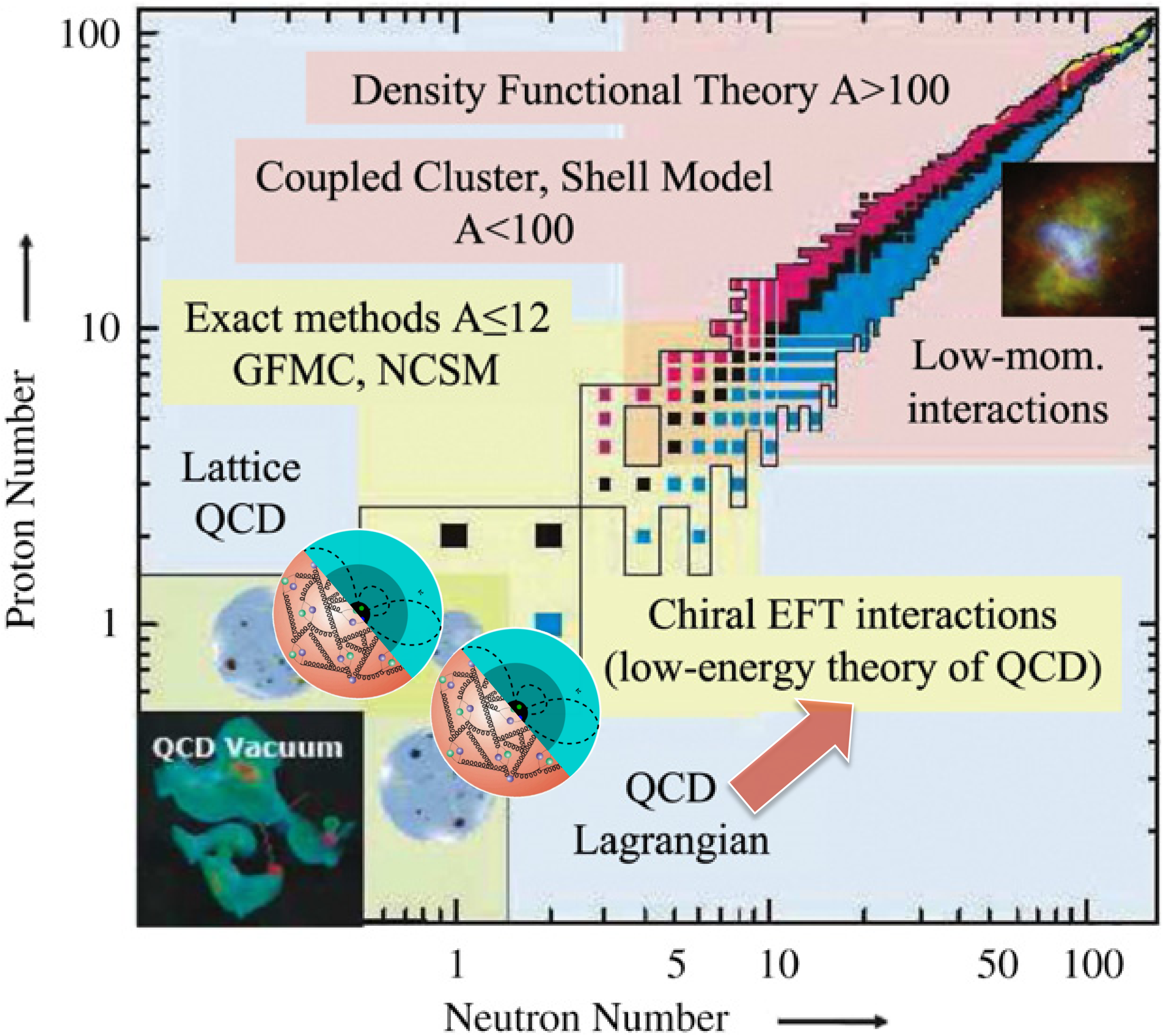
Here we focus on shell model

**How will we approach this problem:**

**QCD → NN (3N) forces → Renormalize → "Solve" many-body problem → Predictions**

# The Challenge of Ab Initio Nuclear Theory

To understand the properties of complex nuclei from first principles




**Nucleon-nucleon interaction**

Some history

Anatomy of an NN interaction

Construction from QCD?

Ideas of Effective Field Theory

Chiral EFT for nuclear forces

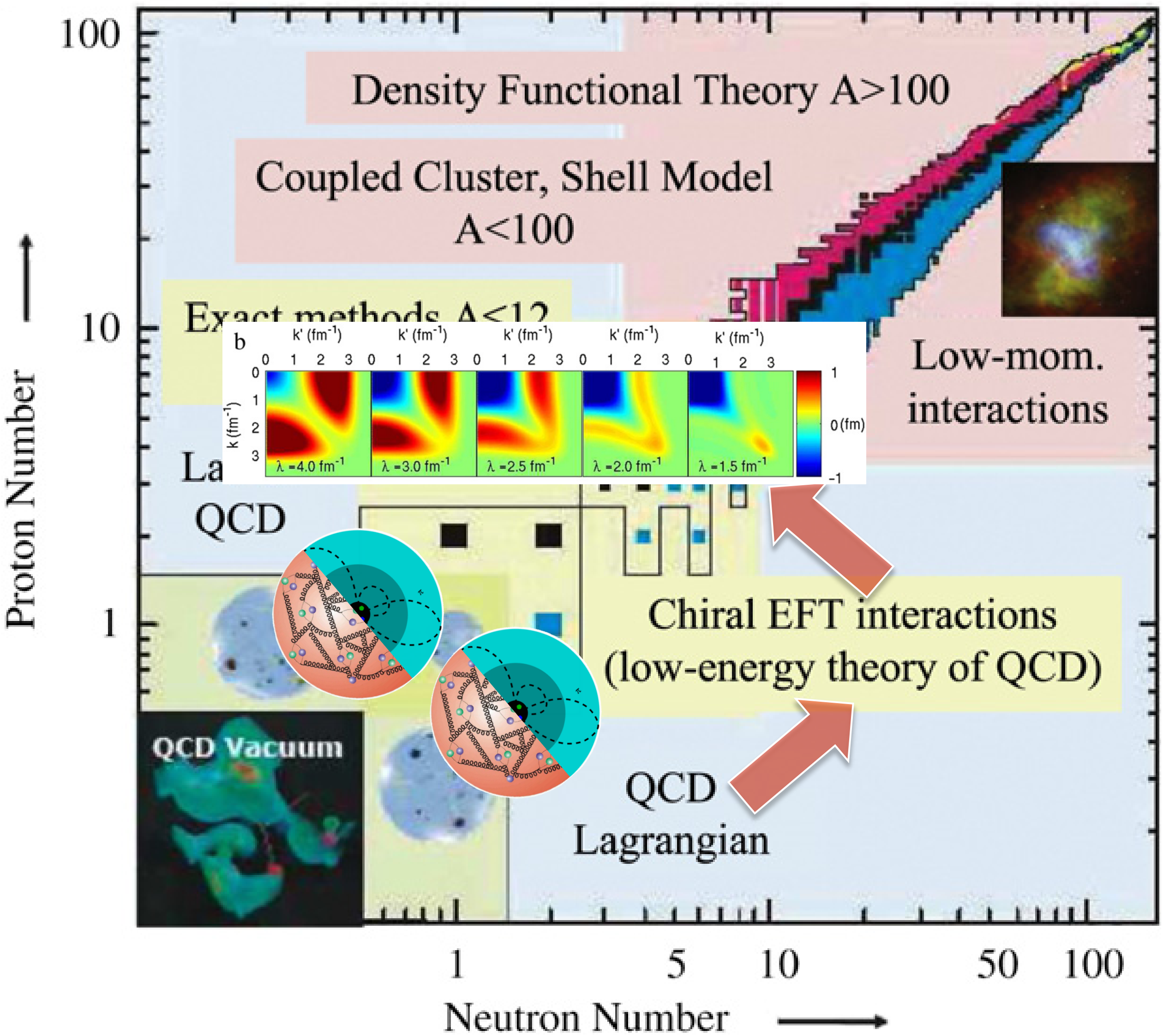
Constraint by data

**How will we approach this problem:**

**QCD → NN (3N) forces → Renormalize → "Solve" many-body problem → Predictions**

# The Challenge of Ab Initio Nuclear Theory

To understand the properties of complex nuclei from first principles




**Renormalizing NN Interactions**

Basic ideas of RG

Low-momentum interactions

Similarity RG interactions

Benefits of low cutoffs

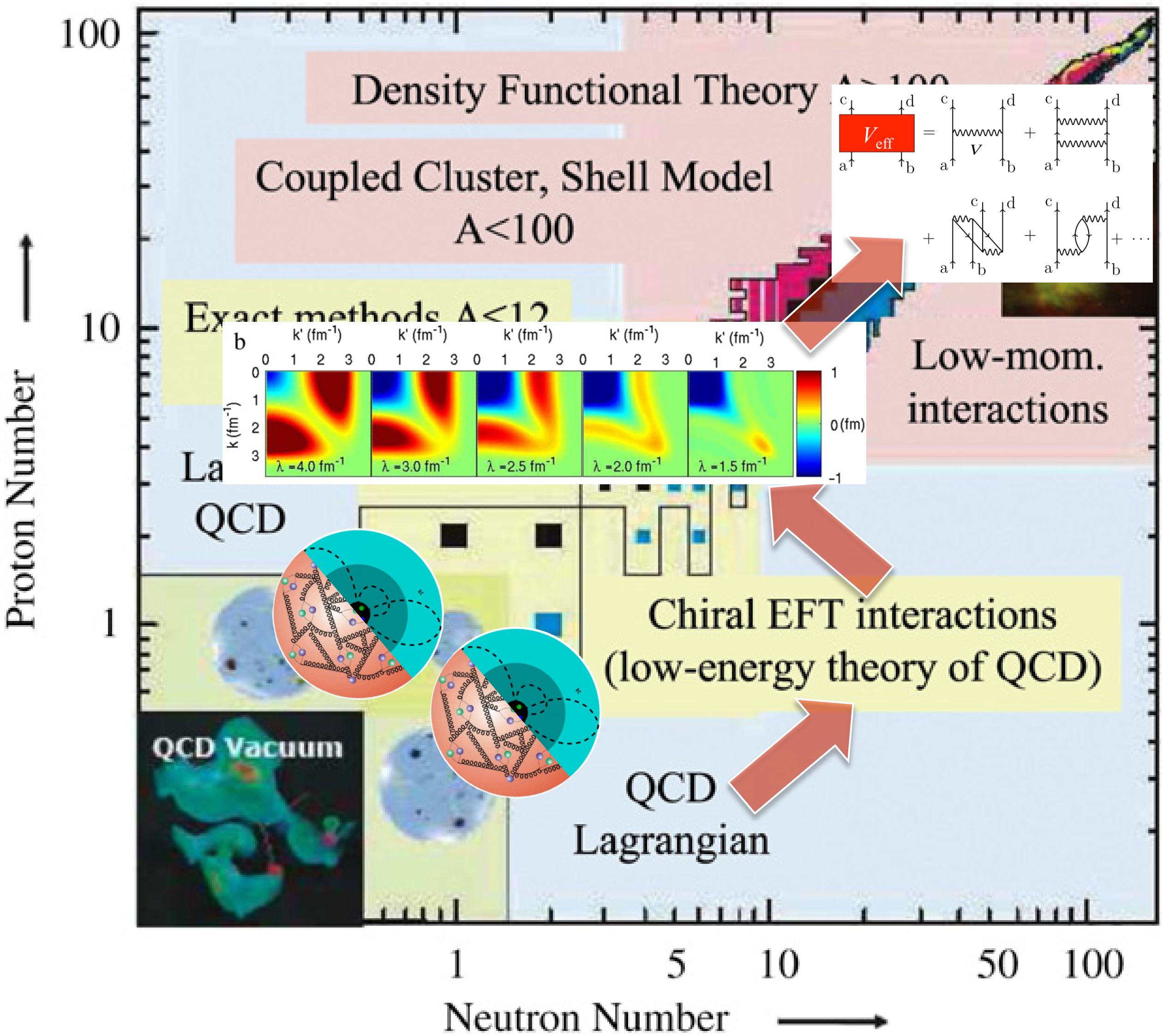
G-matrix renormalization

**How will we approach this problem:**

**QCD → NN (3N) forces → Renormalize → "Solve" many-body problem → Predictions**

# The Challenge of Ab Initio Nuclear Theory

To understand the properties of complex nuclei from first principles




## Microscopic Valence-Space Interactions

Model spaces

Many-body perturbation theory (MBPT)

Calculating effective interaction

In-medium Similarity RG

Monopole part of interaction

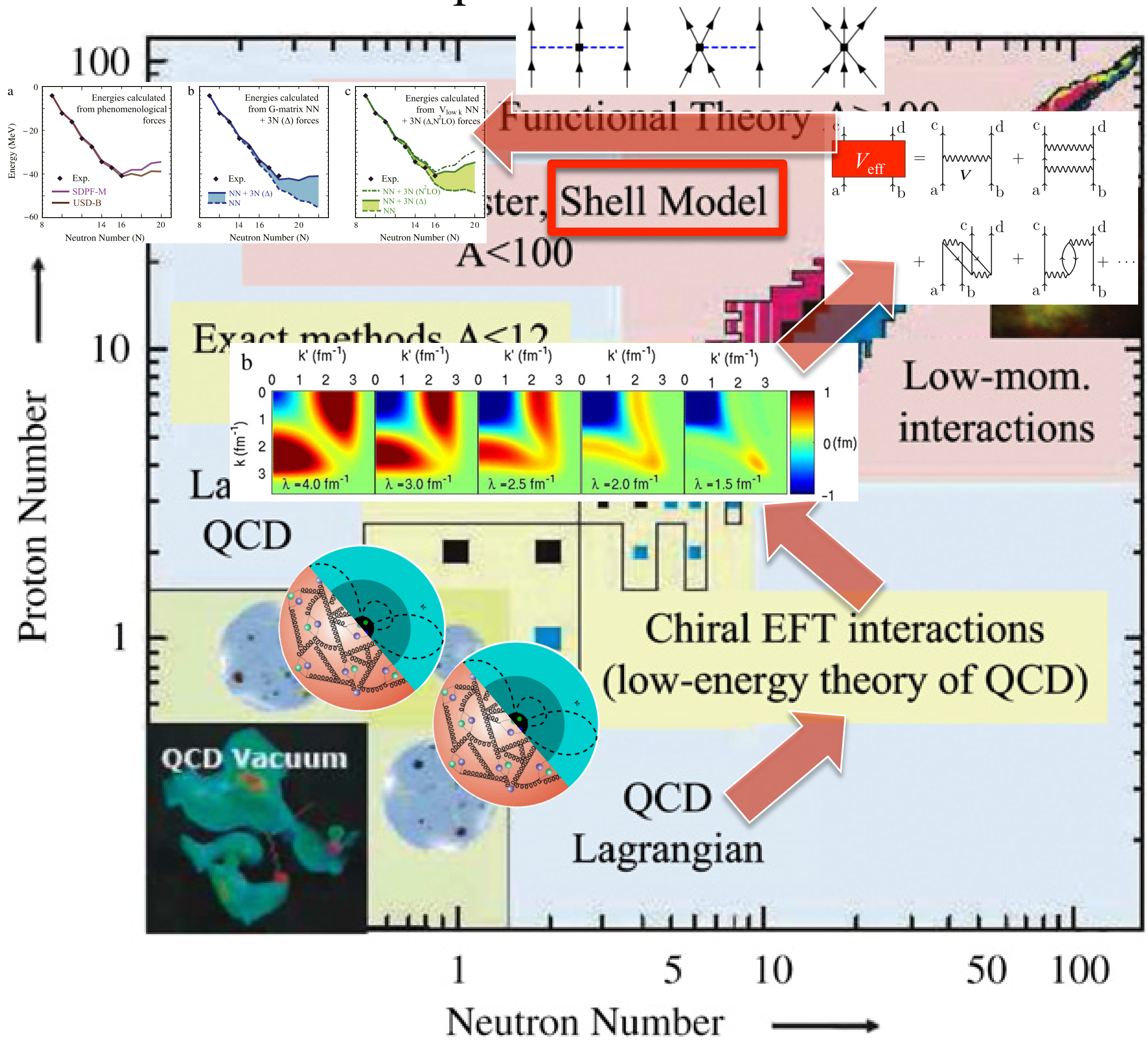
Deficiencies of this approach

**How will we approach this problem:**

**QCD → NN (3N) forces → Renormalize → "Solve" many-body problem → Predictions**

# The Challenge of Ab Initio Nuclear Theory

To understand the properties of complex nuclei from first principles




## Three-Nucleon Forces

Basic ideas – why needed?

3N from chiral EFT

Implementing in shell model

Relation to monopoles

Predictions/new discoveries

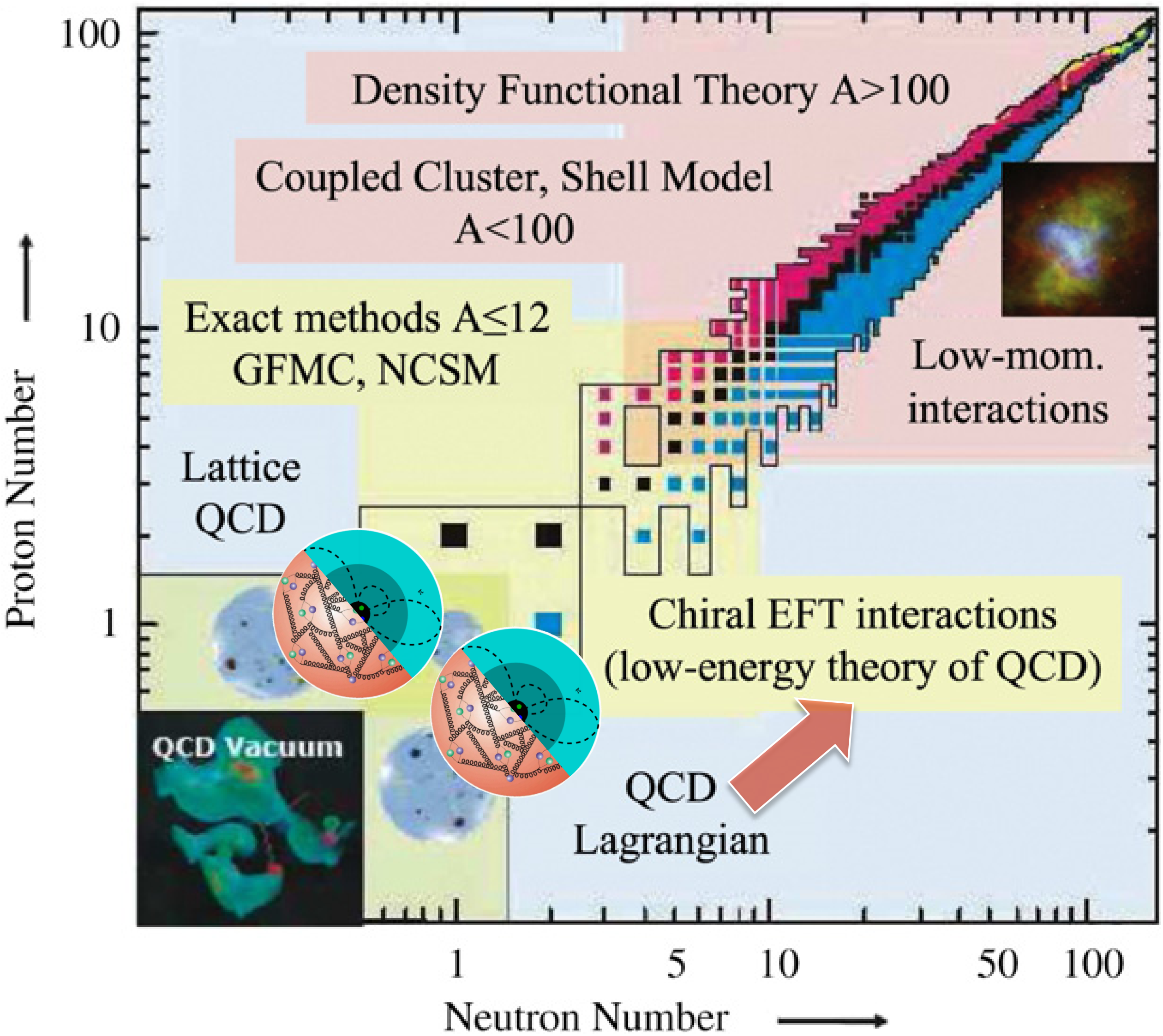
Connections beyond structure

**How will we approach this problem:**

**QCD → NN (3N) forces → Renormalize → "Solve" many-body problem → Predictions**

# Part I: The Nucleon-Nucleon Interaction

To understand the properties of complex nuclei from first principles




**Nucleon-nucleon interaction**

Some history

Anatomy of an NN interaction

Construction from QCD?

Ideas of Effective Field Theory

Chiral EFT for nuclear forces

Constraint by data

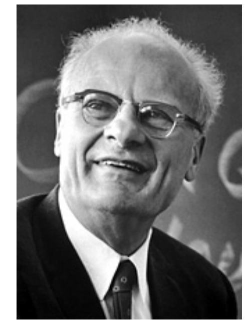
**How will we approach this problem:**

**QCD → NN (3N) forces → Renormalize → Solve many-body problem → Predictions**

# Interaction Between Two Nucleons

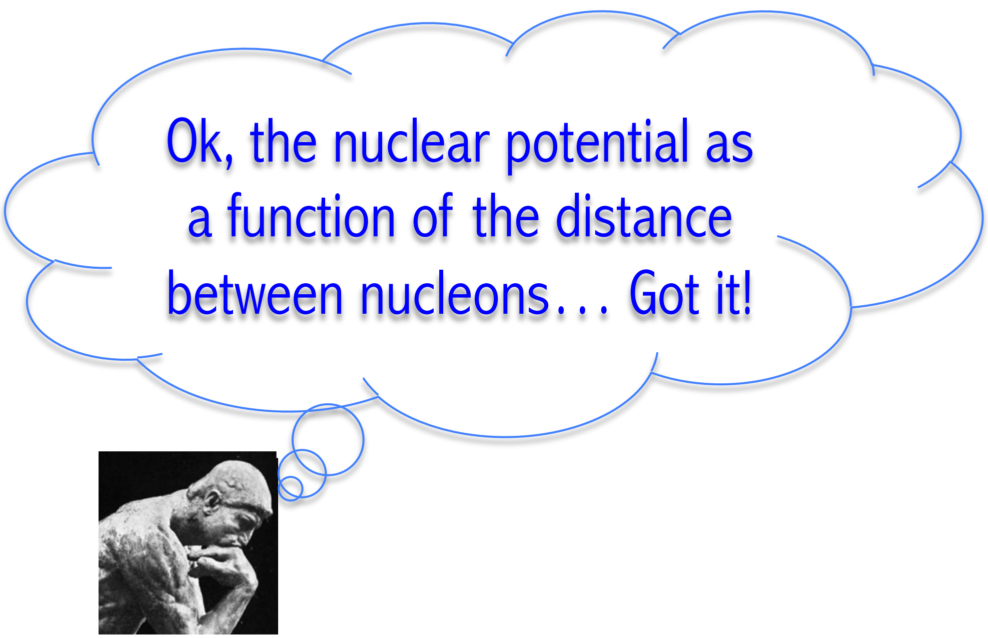
**"In the past quarter century physicists have devoted a huge amount of experimentation and mental labor to this problem – probably more man-hours than have been given to any other scientific question in the history of mankind."**

***–H. Bethe***



***So let's burn a few more man-hours of mental labor on this!***

***To start, think to yourself what this should look like, and write it down…***



# Meson-Exchange Potentials: Yukawa

- First field-theoretical model of nucleon interaction proposed by **Yukawa** 1935
- Postulated nuclear force mediated by (**NEW!**) particle exchange
- Short range ($\sim$1fm) of nuclear force $\Longrightarrow$

# Meson-Exchange Potentials: Yukawa

- First field-theoretical model of nucleon interaction proposed by **Yukawa** 1935
- Postulated nuclear force mediated by (**NEW!**) particle exchange
- Short range (~1fm) of nuclear force $\Longrightarrow$

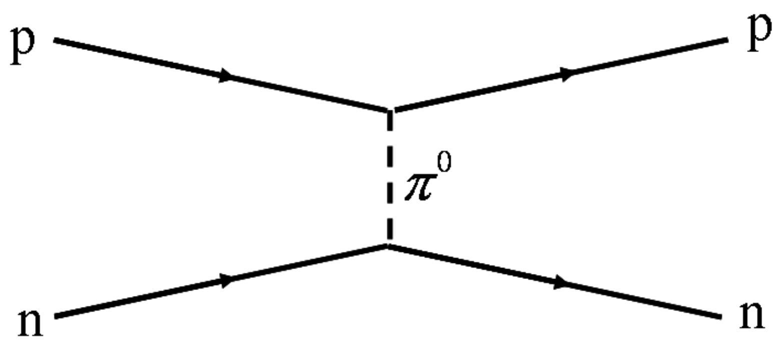
  New particle must be massive: $r \sim 1/m; \quad m =?$

  Hint: $\hbar c \approx 197\,\mathrm{MeV}\cdot\mathrm{fm}$

# Meson-Exchange Potentials: Yukawa

- First field-theoretical model of nucleon interaction proposed by **Yukawa** 1935
- Postulated nuclear force mediated by (**NEW!**) particle exchange
- Short range (~1fm) of nuclear force $\Longrightarrow$

  New particle must be massive: $r \sim 1/m; \quad m = ?$



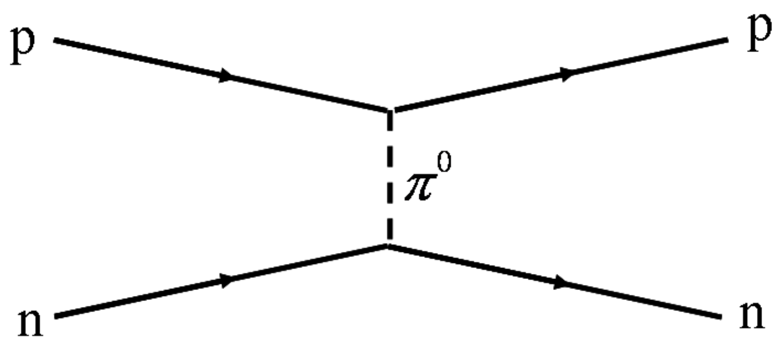

- Pion discovered 1947!

$$V(\vec{r}) = -\frac{f_\pi^2}{m_\pi^2}\left\{\vec{\sigma}_1 \cdot \vec{\sigma}_2 + C_T\left(1 + \frac{3}{m_\alpha r} + \frac{3}{(m_\alpha r)^2}\right) S_{12}(r)\right\}\frac{e^{-m_\pi r}}{m_\pi r}$$

# Meson-Exchange Potentials: Yukawa

- First field-theoretical model of nucleon interaction proposed by **Yukawa** 1935
- Postulated nuclear force mediated by (**NEW!**) particle exchange
- Short range (~1fm) of nuclear force $\Longrightarrow$

  New particle must be massive: $r \sim 1/m; \quad m = ?$




- Pion discovered 1947!

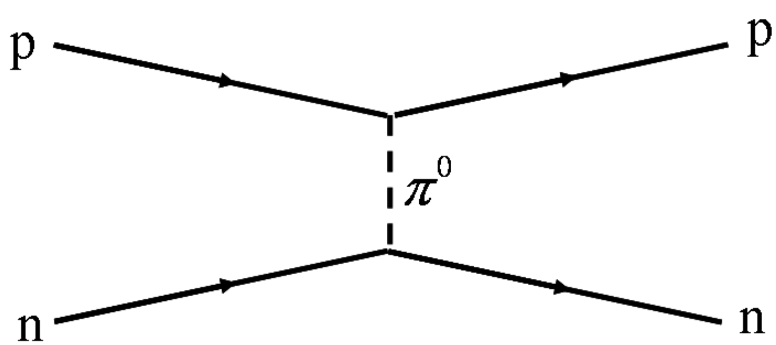
$$V(\vec{r}) = -\frac{f_\pi^2}{m_\pi^2}\left\{\vec{\sigma}_1\cdot\vec{\sigma}_2 + C_T\left(1+\frac{3}{m_\alpha r}+\frac{3}{(m_\alpha r)^2}\right)S_{12}(r)\right\}\frac{e^{-m_\pi r}}{m_\pi r}$$

- **Attractive, "long" range**

# Meson-Exchange Potentials: Yukawa

- First field-theoretical model of nucleon interaction proposed by **Yukawa** 1935
- Postulated nuclear force mediated by (**NEW!**) particle exchange
- Short range (~1fm) of nuclear force $\Longrightarrow$

  New particle must be massive: $r \sim 1/m; \quad m = ?$




- Pion discovered 1947!

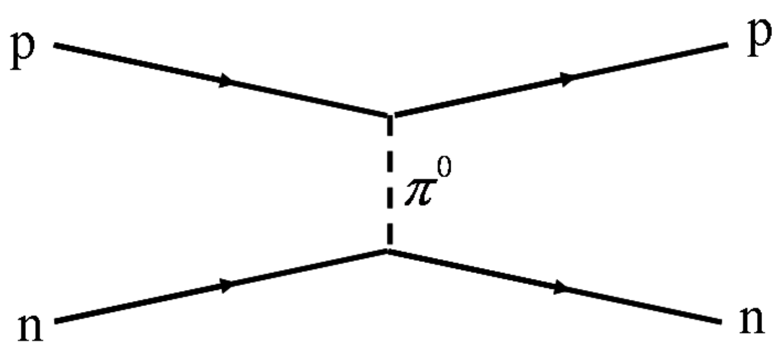
$$V(\vec{r}) = -\frac{f_\pi^2}{m_\pi^2}\left\{\vec{\sigma}_1 \cdot \vec{\sigma}_2 + C_T\left(1 + \frac{3}{m_\alpha r} + \frac{3}{(m_\alpha r)^2}\right) S_{12}(r)\right\}\frac{e^{-m_\pi r}}{m_\pi r}$$

- Attractive, "long" range, **spin dependent**

# Meson-Exchange Potentials: Yukawa

- First field-theoretical model of nucleon interaction proposed by **Yukawa** 1935
- Postulated nuclear force mediated by (**NEW!**) particle exchange
- Short range (~1fm) of nuclear force $\Longrightarrow$

  New particle must be massive: $r \sim 1/m; \quad m = ?$




- Pion discovered 1947!

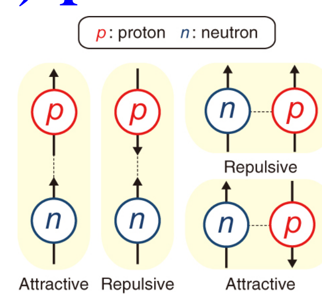
$$V(\vec{r}) = -\frac{f_\pi^2}{m_\pi^2}\left\{\vec{\sigma}_1 \cdot \vec{\sigma}_2 + C_T\left(1 + \frac{3}{m_\alpha r} + \frac{3}{(m_\alpha r)^2}\right) S_{12}(r)\right\} \frac{e^{-m_\pi r}}{m_\pi r}$$

- Attractive, "long" range, spin dependent, **non-central (tensor) part**

  Depends on spin, isospin, orientation of nucleons

  Does not conserve $L^2$, $S^2$, but does conserve parity

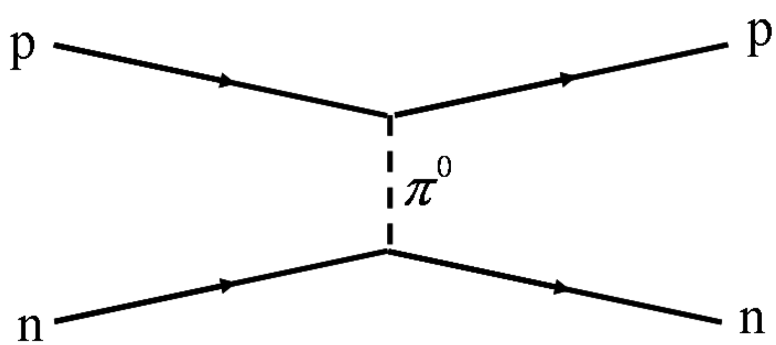
  $\Longrightarrow$ Mixes different L states (but only differing by 2 units)



# Meson-Exchange Potentials: Yukawa

- First field-theoretical model of nucleon interaction proposed by **Yukawa** 1935
- Postulated nuclear force mediated by (**NEW!**) particle exchange
- Short range (~1fm) of nuclear force $\Longrightarrow$

  New particle must be massive: $r \sim 1/m; \quad m = ?$



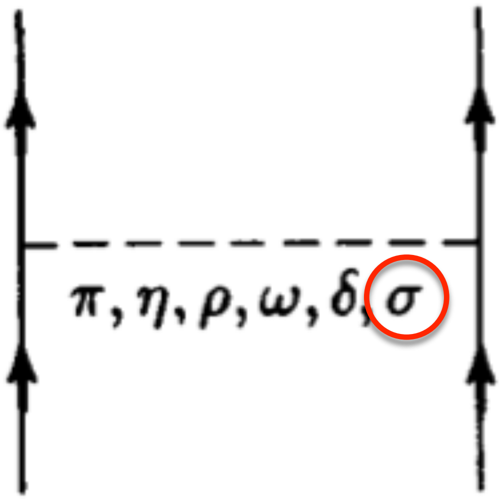

- Pion discovered 1947!

$$V(\vec{r}) = -\frac{f_\pi^2}{m_\pi^2}\left\{\vec{\sigma}_1 \cdot \vec{\sigma}_2 + C_T \left(1 + \frac{3}{m_\alpha r} + \frac{3}{(m_\alpha r)^2}\right) S_{12}(r)\right\} \frac{e^{-m_\pi r}}{m_\pi r}$$

- **Attractive, "long" range, spin dependent, non-central (tensor) part**
- Successful in explaining scattering data, deuteron
- One pion is good, therefore more pions are better…
- Advanced to multi-pion theories in 1950's – **FAILED! Now what??**

# One-Boson Exchange Potentials

- Heavy mesons discovered in late 1950s – formed basis for new theories
- Intermediate range – **attractive central, spin-orbit**



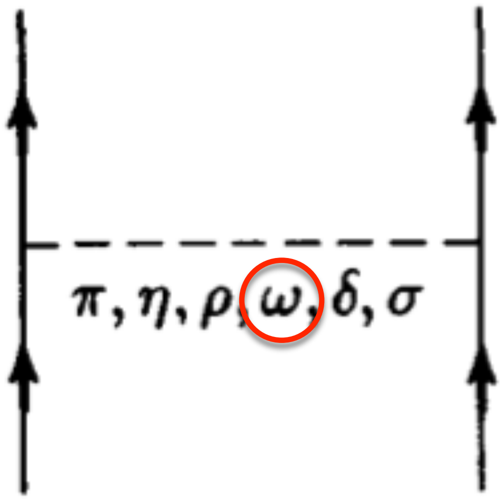

$$V^{\sigma} = g_{\sigma NN}^{2} \frac{1}{\mathbf{k}^2 + m_\sigma^2} \left( -1 + \frac{\mathbf{q}^2}{2M_N^2} - \frac{\mathbf{k}^2}{8M_N^2} - \frac{\vec{L}\cdot\vec{S}}{2M_N^2} \right)$$

$$\vec{q}_i \equiv \vec{p}_i^{\,\prime} - \vec{p}_i \qquad \vec{k}_i \equiv \frac{1}{2}\left(\vec{p}_i^{\,\prime} + \vec{p}_i\right)$$

| Baryons | Mass (MeV) | Mesons | Mass (MeV) |
|---|---|---|---|
| p, n | 938.926 | $\pi$ | 138.03 |
| $\Lambda$ | 1116.0 | $\eta$ | 548.8 |
| $\Sigma$ | 1197.3 | $\sigma$ | $\approx$ 550.0 |
| $\Delta$ | 1232.0 | $\rho$ | 770 |
| $\Sigma^*$ | 1385.0 | $\omega$ | 782.6 |
| | | $\delta$ | 983.0 |
| | | K | 495.8 |
| | | K* | 895.0 |

# One-Boson Exchange Potentials

- Heavy mesons discovered in late 1950s – formed basis for new theories
- Short range; **repulsive central force, attractive spin-orbit**

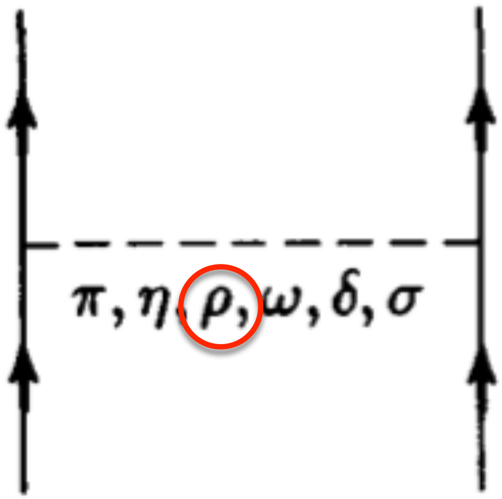



$$V^{\omega} = g^2_{\omega NN} \frac{1}{\mathbf{k}^2 + m_\omega^2} \left( 1 - 3 \frac{\vec{L} \cdot \vec{S}}{2 M_N^2} \right)$$

| Baryons | Mass (MeV) | Mesons | Mass (MeV) |
|---|---|---|---|
| p, n | 938.926 | $\pi$ | 138.03 |
| Λ | 1116.0 | $\eta$ | 548.8 |
| Σ | 1197.3 | $\sigma$ | ≈ 550.0 |
| Δ | 1232.0 | $\rho$ | 770 |
| Σ* | 1385.0 | $\omega$ | 782.6 |
| | | $\delta$ | 983.0 |
| | | K | 495.8 |
| | | K* | 895.0 |

# One-Boson Exchange Potentials

- Heavy mesons discovered in late 1950s – formed basis for new theories
- Short range; **tensor force opposite sign of one-pion exchange**




$$V^{\rho} = g_{\rho NN}^2 \frac{\mathbf{k}^2}{\mathbf{k}^2 + m_\rho^2} \left( -2\ \vec{\sigma}_1 \cdot \vec{\sigma}_2 + S_{12}(\hat{k}) \right) \vec{\tau}_1 \cdot \vec{\tau}_2$$

| Baryons | Mass (MeV) | Mesons | Mass (MeV) |
|---|---|---|---|
| p, n | 938.926 | $\pi$ | 138.03 |
| $\Lambda$ | 1116.0 | $\eta$ | 548.8 |
| $\Sigma$ | 1197.3 | $\sigma$ | ≈ 550.0 |
| $\Delta$ | 1232.0 | $\rho$ | 770 |
| $\Sigma^*$ | 1385.0 | $\omega$ | 782.6 |
| | | $\delta$ | 983.0 |
| | | K | 495.8 |
| | | $K^*$ | 895.0 |

# Parameterizing the NN Interaction

Starting from any NN-interaction, first solve:

Lipmann-Schwinger scattering T-matrix equation:

$$T^{\alpha}_{ll'}(k,k';K) = V^{\alpha}_{ll'}(k,k') + \frac{2}{\pi}\sum_{l''}\int_0^{\infty} q^2 \mathrm{d}q\, V^{\alpha}_{ll''}(k,q)\frac{q}{k^2-q^2+i\varepsilon}T^{\alpha}_{l''l'}(q,k';K)$$

where $T^{\alpha}_{ll'}(k,k';K) = \langle kK, lL; JST \,|\, T \,|\, k'K, l'L; JST\rangle$

Parameterized in partial waves α – in **relative/center of mass frame (K,L)**

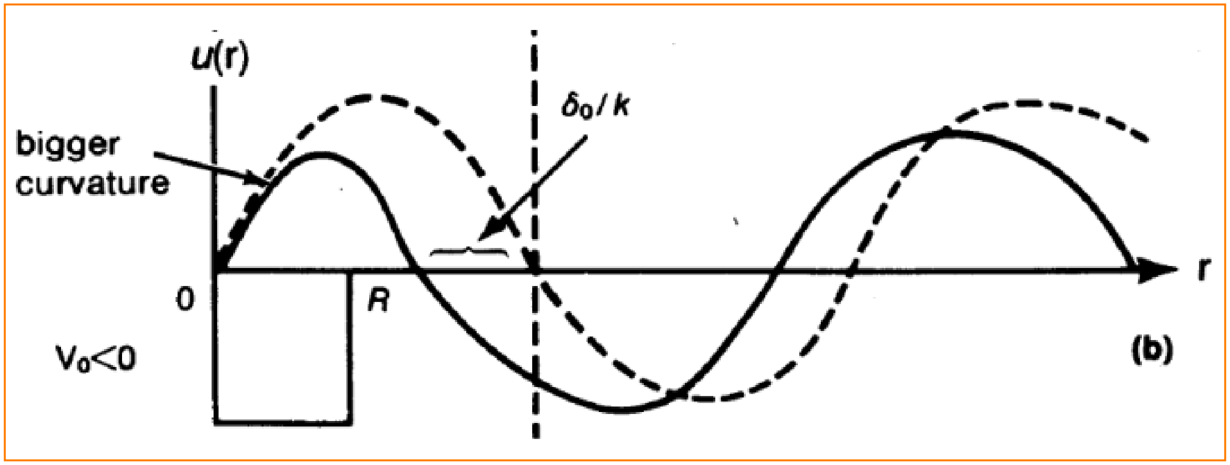
$$\tan\delta(k) = -kT(k,k)$$

**Fully-on-shell *T*-matrix** directly related to experimental data

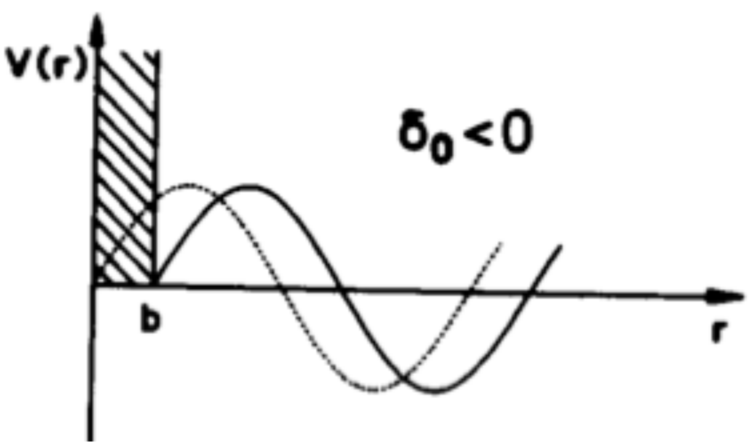
# Constraining NN Scattering Phase Shifts

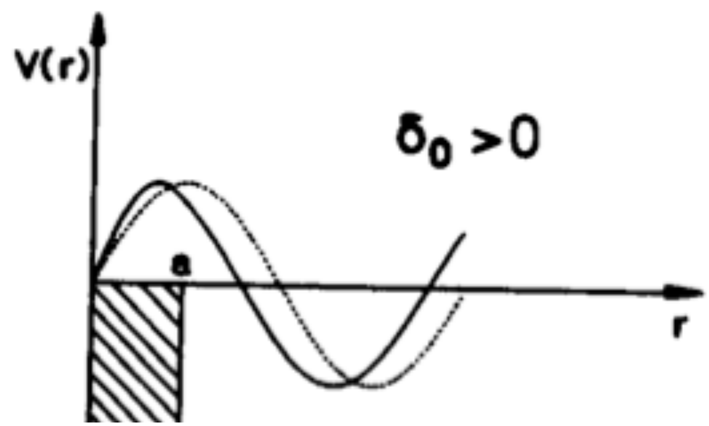
Phase shift is a function of relative momentum $k$; Figure shows $s$-wave

Scattering from an attractive well potential




Scattering from repulsive core: phase shift opposite sign



# Parameterizing the NN Interaction

Starting from any NN-interaction, first solve:

Lipmann-Schwinger scattering T-matrix equation:

$$T^{\alpha}_{ll'}(k,k';K) = V^{\alpha}_{ll'}(k,k') + \frac{2}{\pi}\sum_{l''}\int_0^{\infty} q^2 \mathrm{d}q\, V^{\alpha}_{ll''}(k,q)\frac{q}{k^2-q^2+i\varepsilon}T^{\alpha}_{l''l'}(q,k';K)$$

where $T^{\alpha}_{ll'}(k,k';K) = \langle kK, lL; JST \mid T \mid k'K, l'L; JST\rangle$

Parameterized in partial waves α – in relative/center of mass frame (K,L)

$$\tan\delta(k) = -kT(k,k)$$

Fully-on-shell $T$-matrix directly related to experimental data

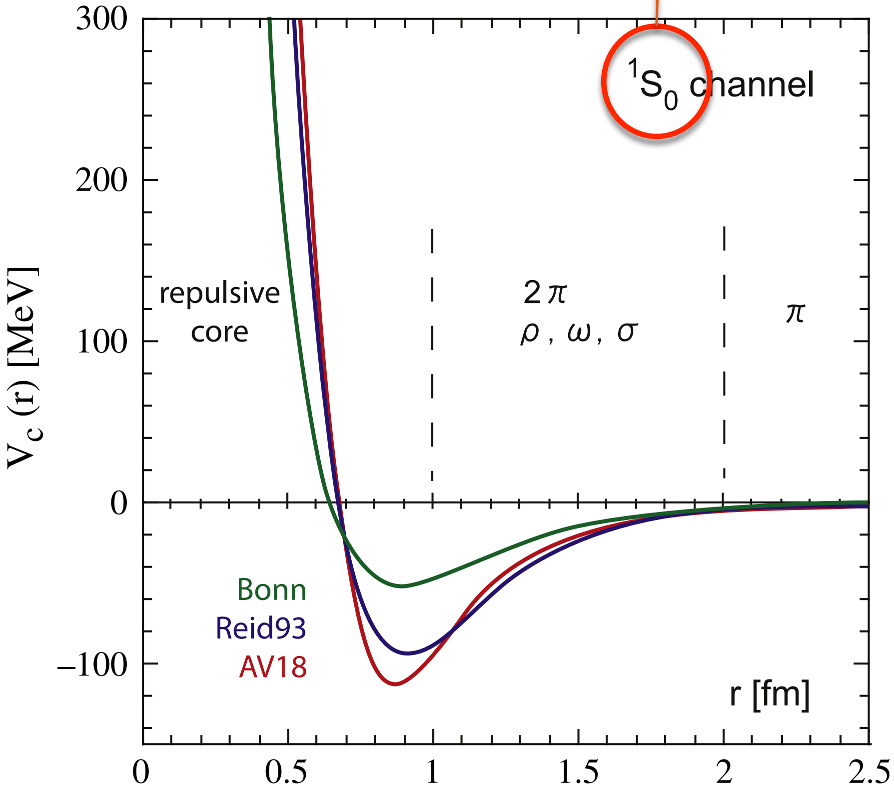
Note intermediate momentum allowed to infinity (but cutoff by regulators)

**Coupling of low-to-high momentum in $V$**

# Form of NN Interactions

Textbook nuclear potentials in coordinate (**r**) space (distance between nucleons)

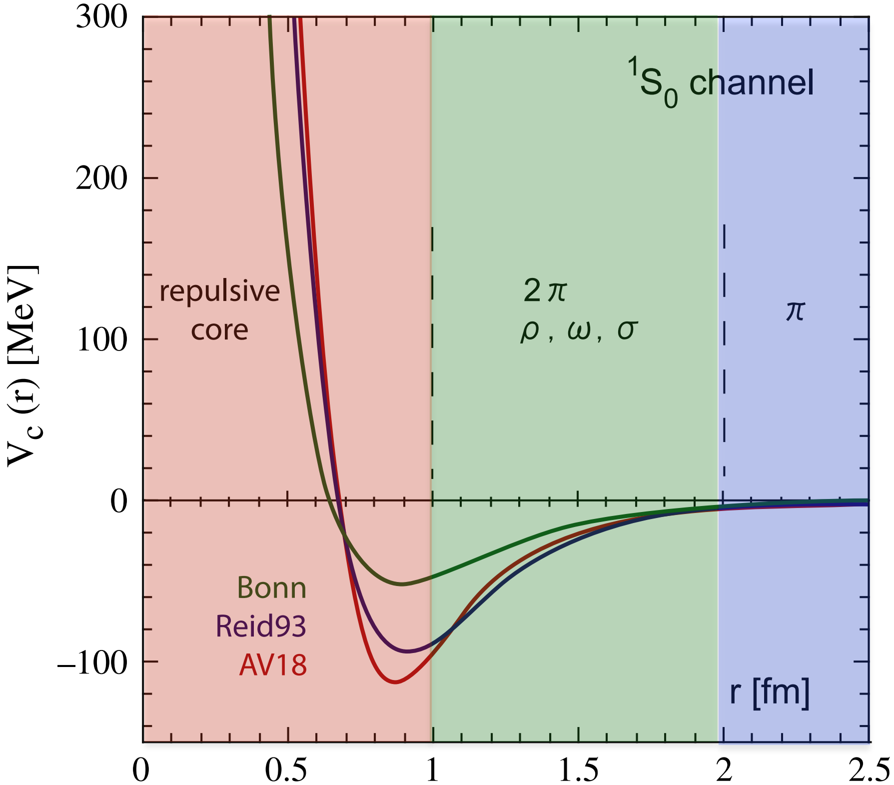
$$^{2S+1}L_J$$



# Form of NN Interactions

Textbook nuclear potentials in coordinate (**r**) space (distance between nucleons)

**Hard core**, **intermediate-range 2π**, **long-range 1π exchange (OPE)**



# Form of NN Interactions

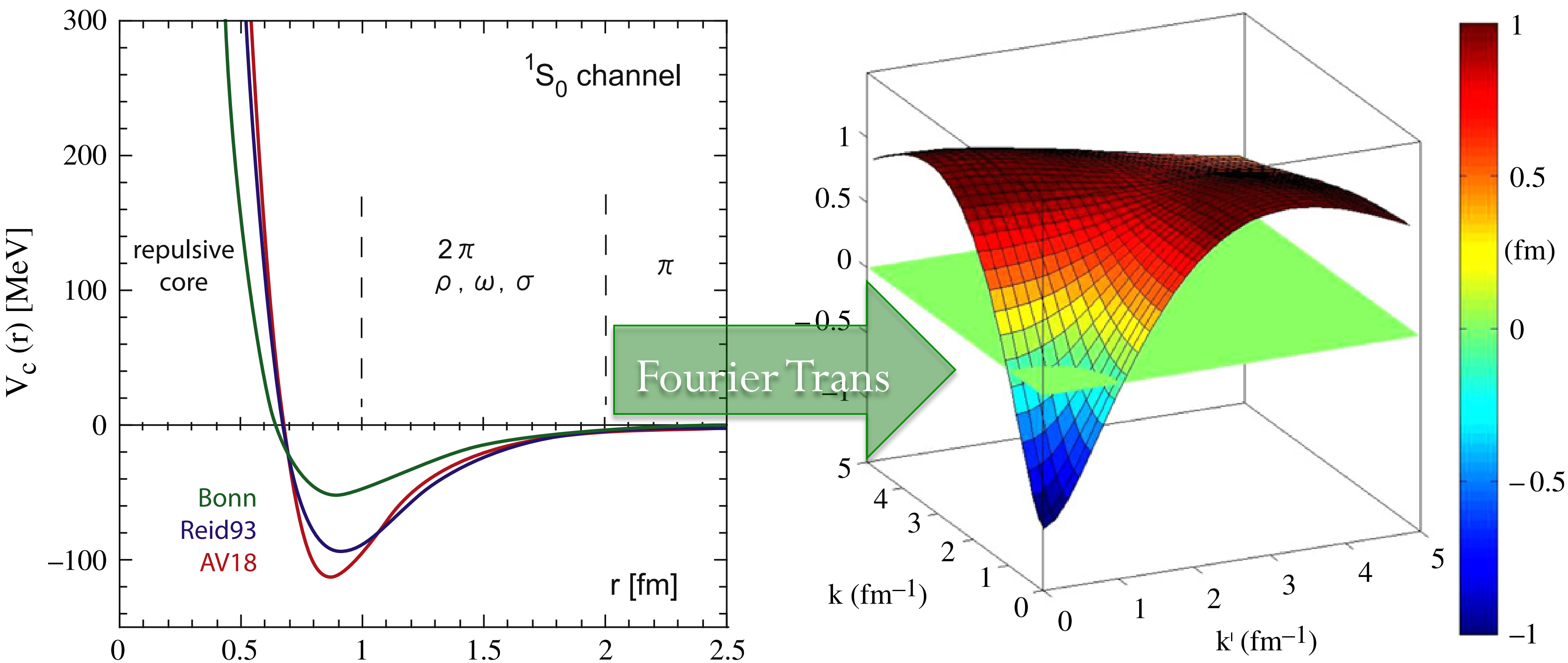
Textbook nuclear potentials in coordinate (**r**) space (distance between nucleons)

Hard core, intermediate-range 2**π**, long-range 1**π** exchange

Transform to momentum space via **Fourier Transformation**

**Strong high-momentum repulsion**, **low-momentum attraction**

$$V_l(k,k') = \frac{2}{\pi}\int_0^\infty r^2 \mathrm{d}r\, j_l(kr)V(r)j_l(k'r)$$



# Form of NN Interactions

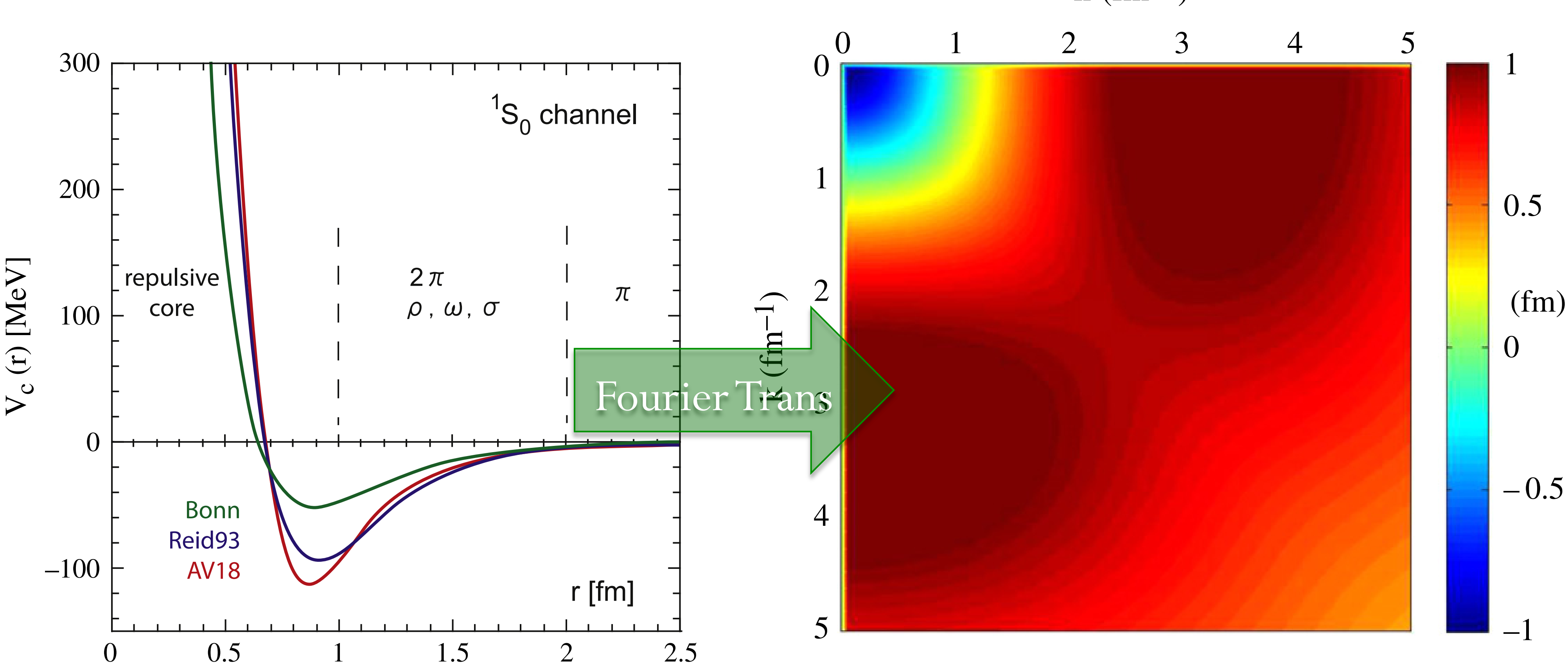
Textbook nuclear potentials in coordinate (**r**) space (distance between nucleons)

Hard core, intermediate-range 2**π**, long-range 1**π** exchange

Transform to momentum space via **Fourier Transformation**

**Strong high-momentum repulsion**, **low-momentum attraction**

$$V_l(k,k') = \frac{2}{\pi}\int_0^\infty r^2 \mathrm{d}r\, j_l(kr)V(r)j_l(k'r)$$



# Form of NN Interactions

Textbook nuclear potentials in coordinate (**r**) space (distance between nucleons)

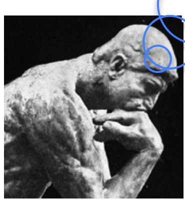
Hard core, intermediate-range 2π, long-range 1π exchange

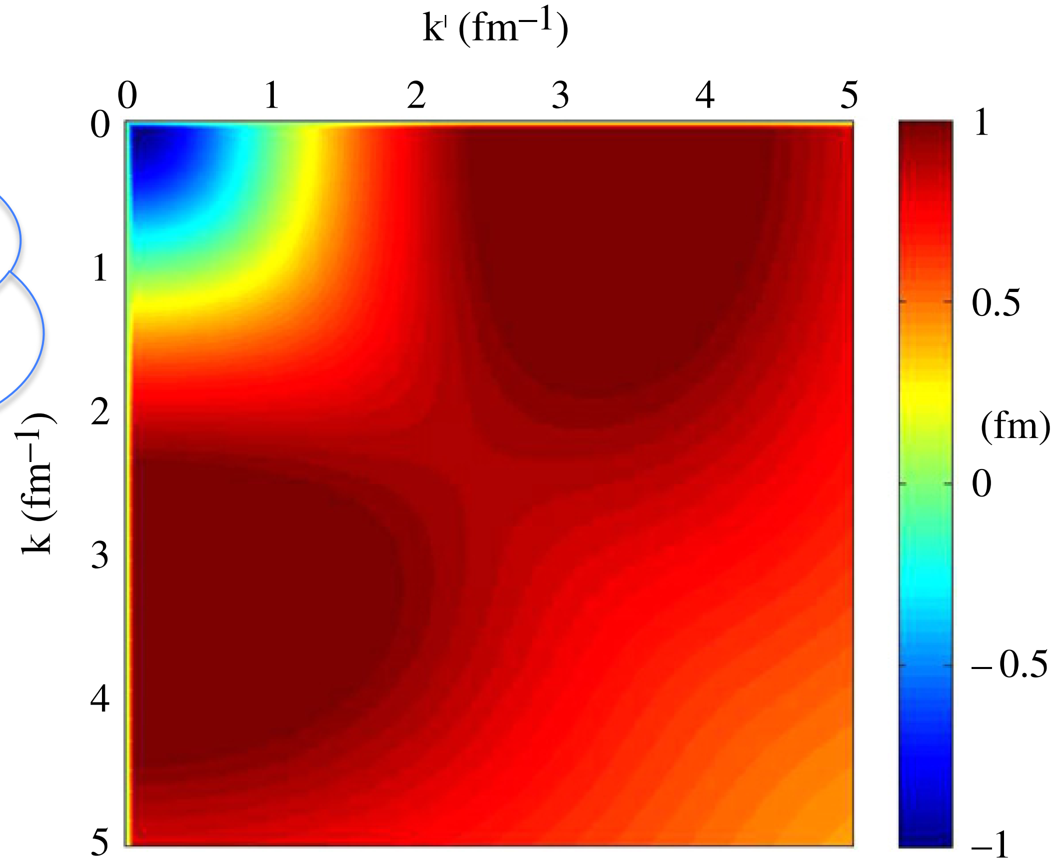
Transform to momentum space via **Fourier Transformation**

**Strong high-momentum repulsion**, **low-momentum attraction**

$$V_l(k,k') = \frac{2}{\pi}\int_0^{\infty} r^2 \mathrm{d}r\, j_l(kr)V(r)j_l(k'r)$$

Wait a minute… these potentials can't really go to zero range/infinitely high energies; that would be QCD?



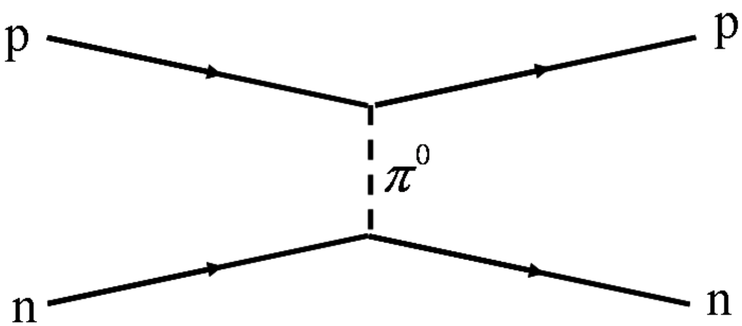


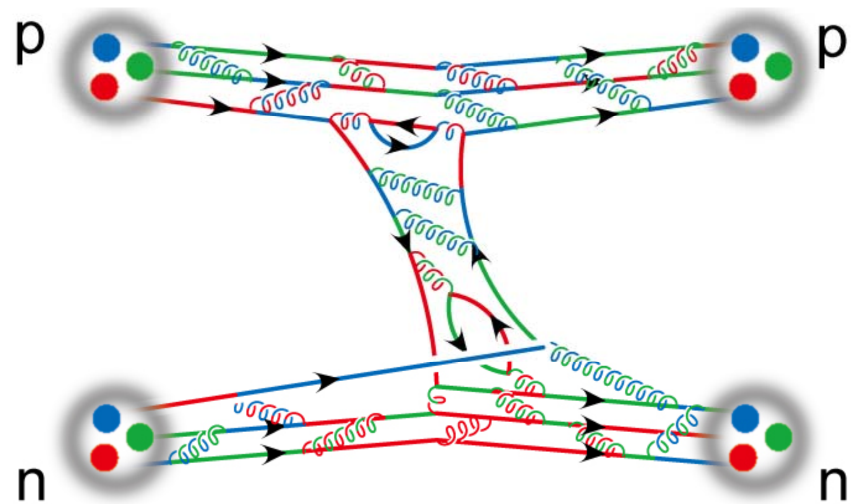
# NN Interaction from QCD?

Meson exchange in principle described in Quantum Chromodynamics (QCD)

Low-energy region non-perturbative – treat in the context of **Lattice QCD**

Directly from QCD Lagrangian, solve numerically on discretized space-time



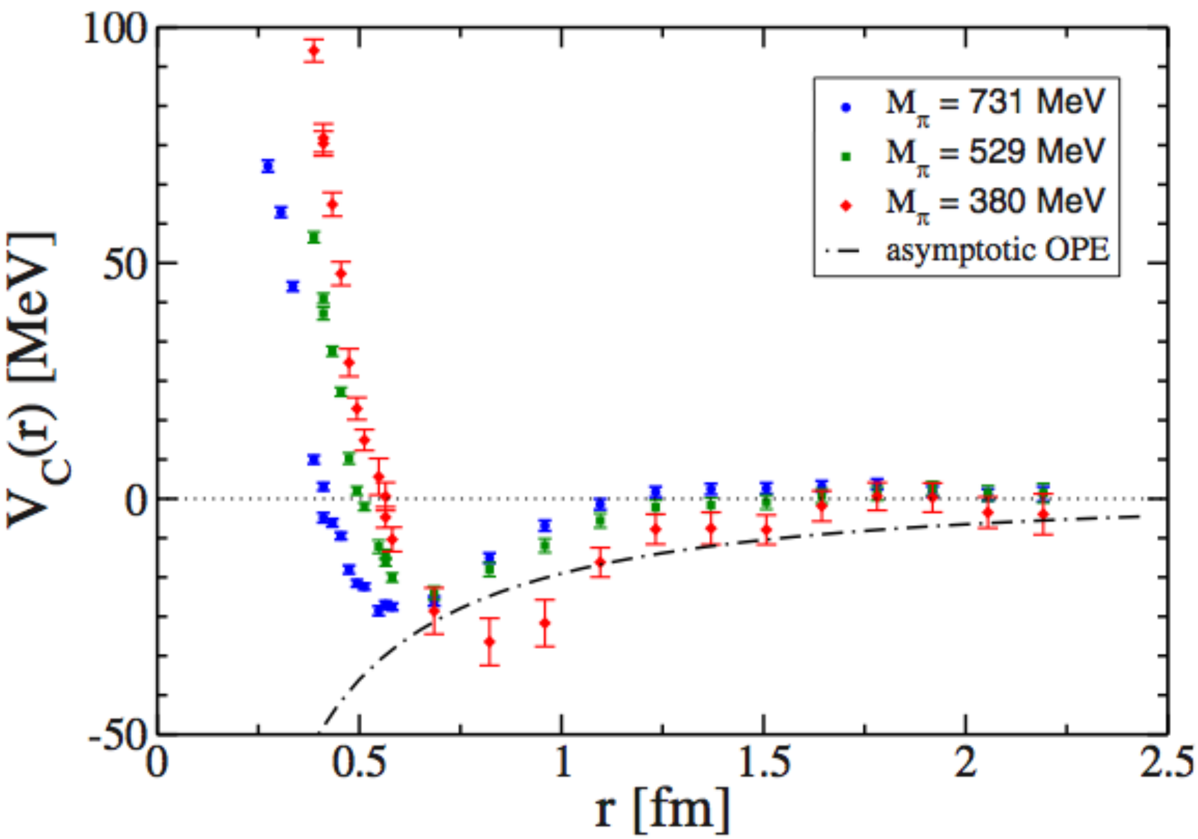



# NN Interaction from QCD?

Meson exchange described in QCD

Low-energy region non-perturbative – treat in the context of **Lattice QCD**

Directly from QCD Lagrangian, solve numerically on discretized space-time



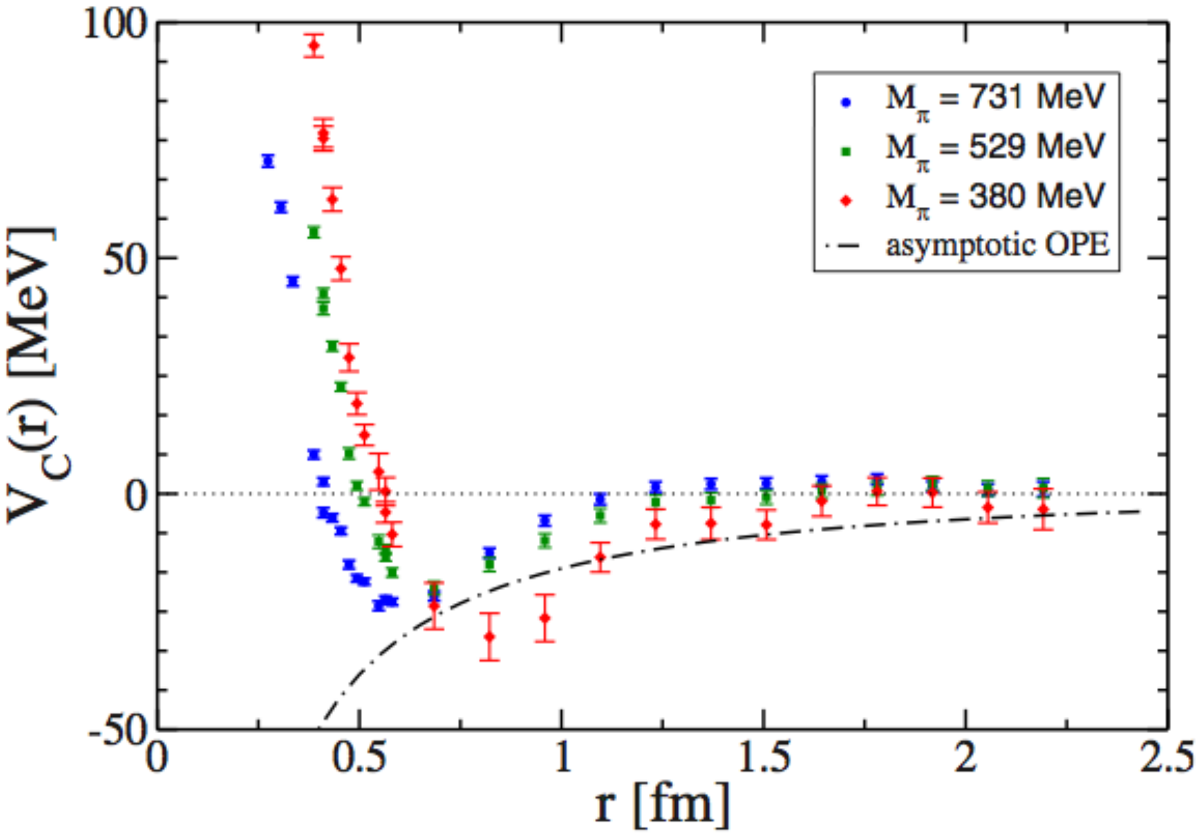

Lattice results give long-range OPE tail, hard core

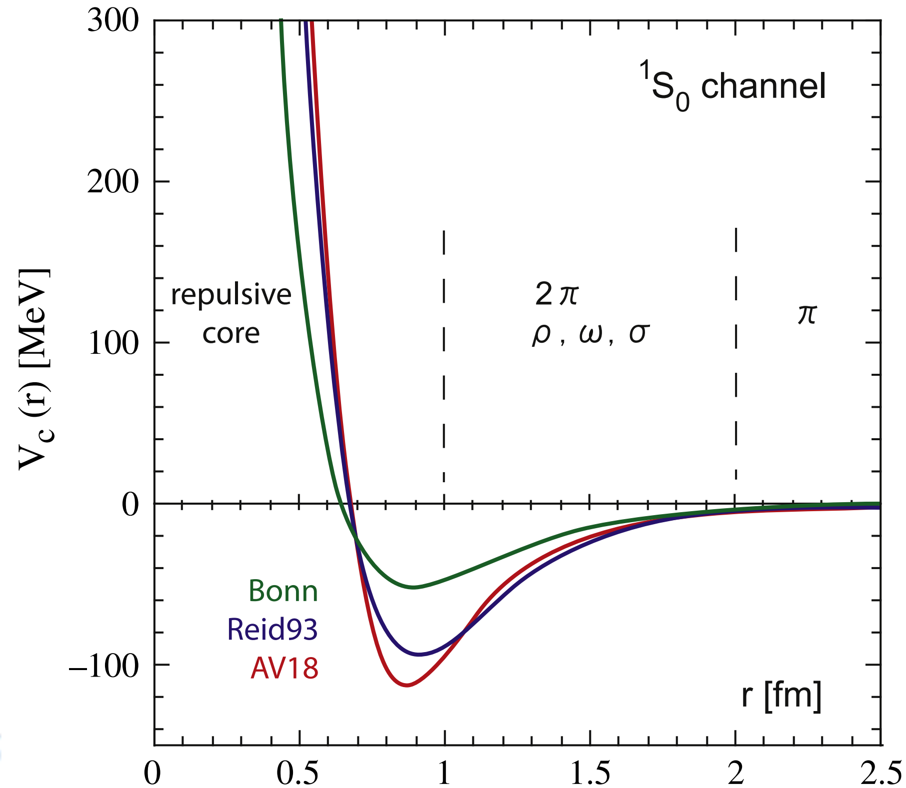
# NN Interaction from QCD?

Meson exchange described in QCD

Low-energy region non-perturbative – treat in the context of **Lattice QCD**

Directly from QCD Lagrangian, solve numerically on discretized space-time



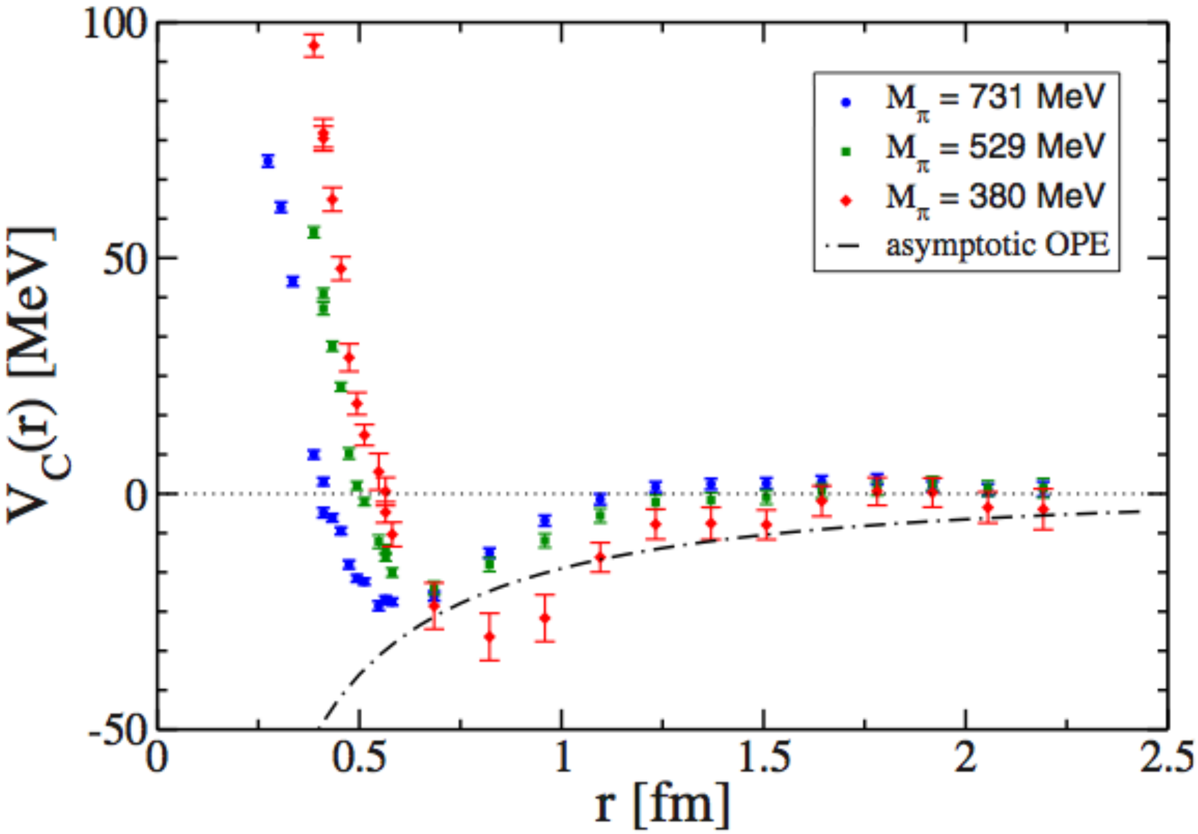

Lattice results give long-range OPE tail, hard core

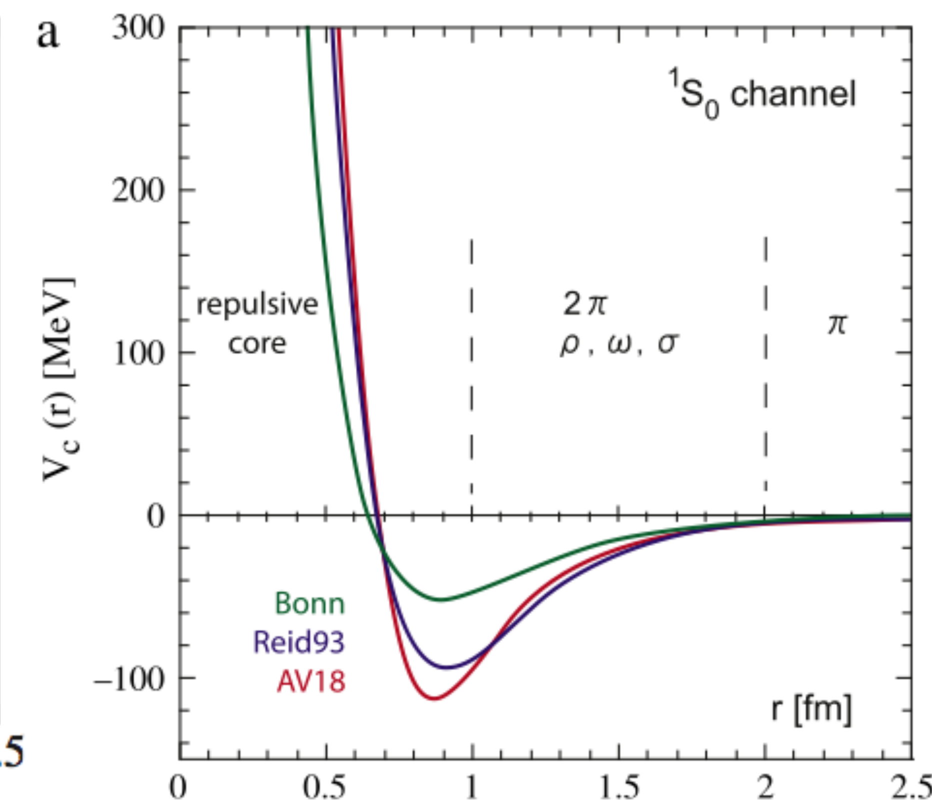
Not yet to physical pion mass – work in progress – so we're done, right?

# Unique NN Potential?

What does this tell us in our quest for an NN-potential?

Expected form seems to be confirmed by QCD



# OBE Potentials: Summary/Problems

**First generation** (1960-1990): Paris, Reid, Bonn-A,B,C $\chi^2/\mathrm{dof} \approx 2$

**High-precision potentials** (1990s): Focus on precision ~40 parameters fit NN data

ArgonneV18, Reid93, Nijmegen, CD-Bonn $\chi^2/\mathrm{dof} \approx 1$

NN problem "solved" !!

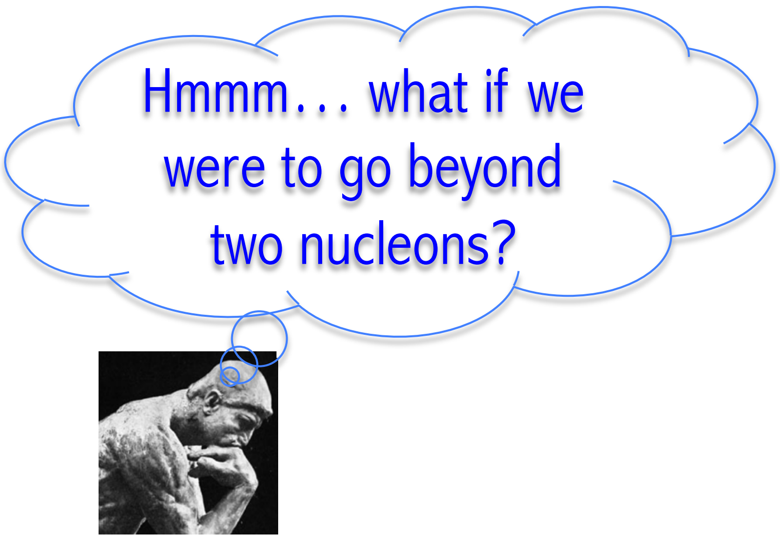
# OBE Potentials: Summary/Problems

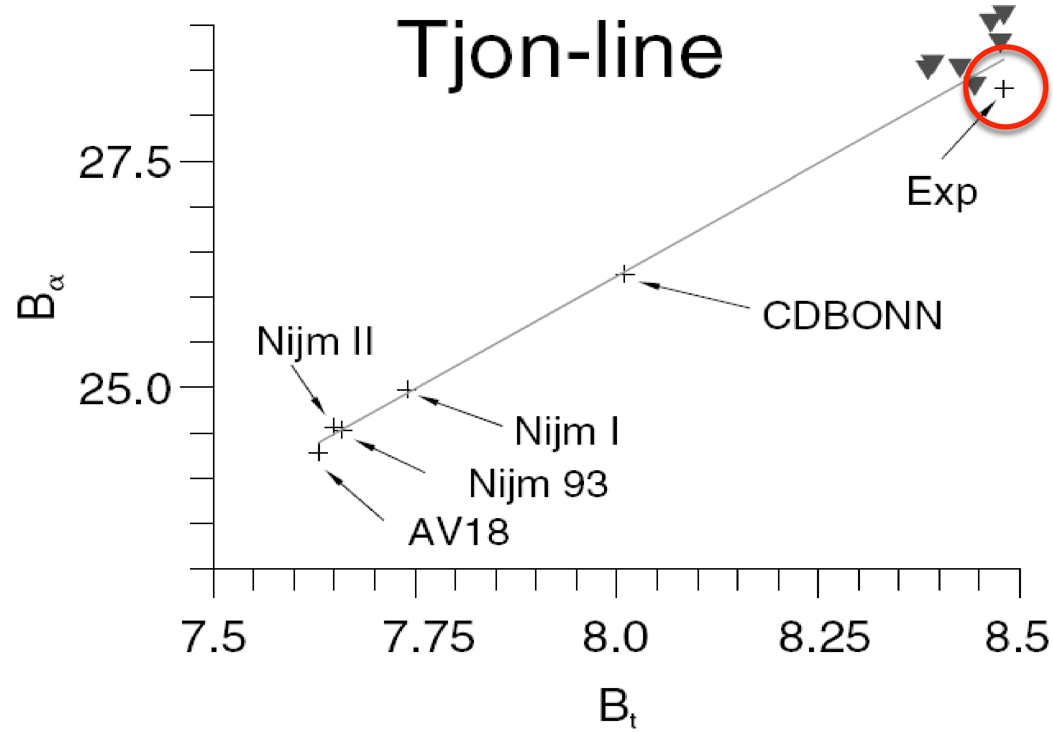
**First generation** (1960-1990): Paris, Reid, Bonn-A,B,C $\chi^2/\mathrm{dof} \approx 2$

**High-precision potentials** (1990s): Focus on precision ~40 parameters fit NN data

ArgonneV18, Reid93, Nijmegen, CD-Bonn $\chi^2/\mathrm{dof} \approx 1$

NN problem "solved" !!



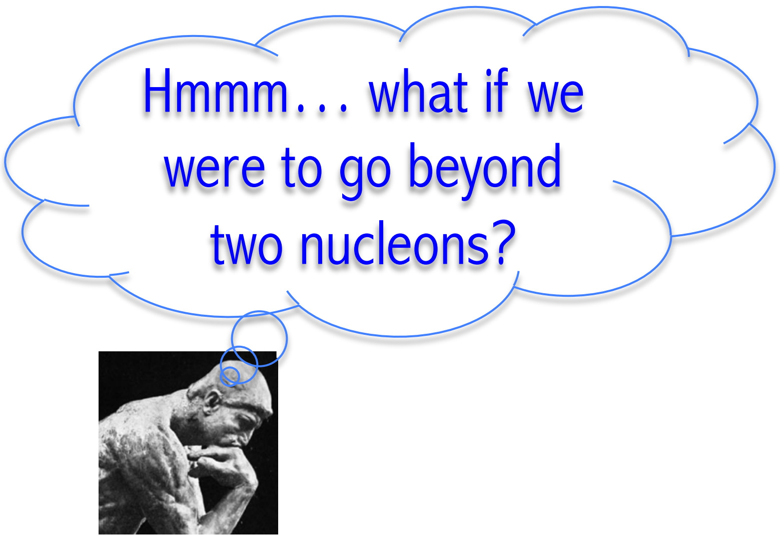
# OBE Potentials: Summary/Problems

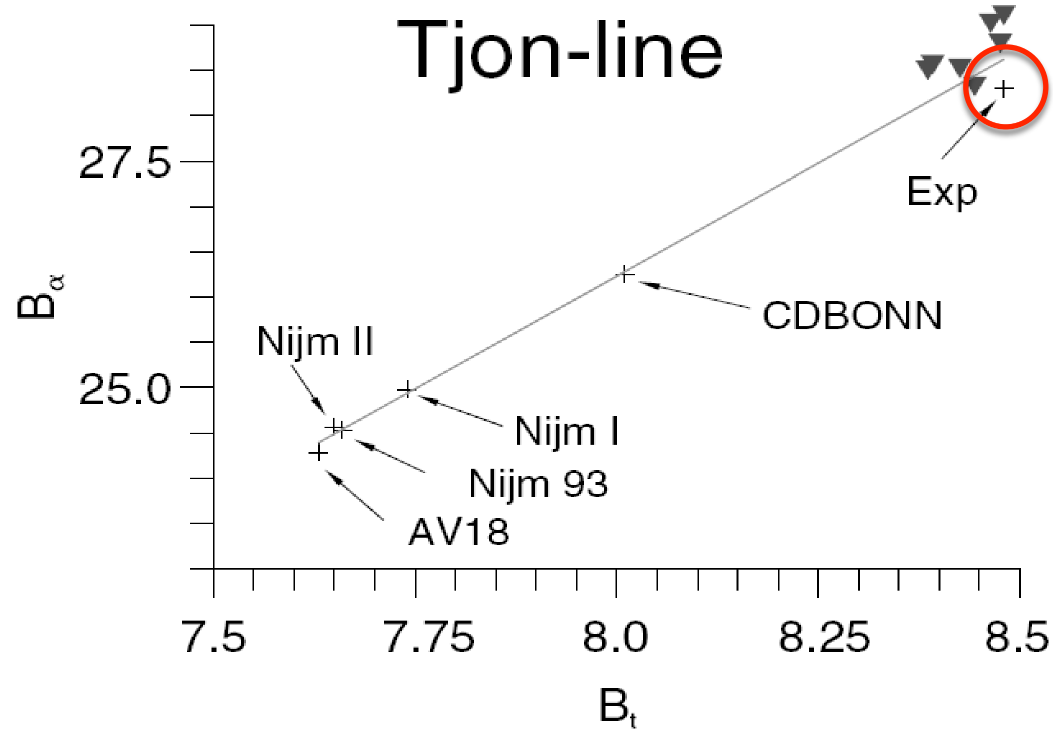
**First generation** (1960-1990): Paris, Reid, Bonn-A,B,C $\chi^2/\mathrm{dof} \approx 2$

**High-precision potentials** (1990s): Focus on precision ~40 parameters fit NN data

ArgonneV18, Reid93, Nijmegen, CD-Bonn $\chi^2/\mathrm{dof} \approx 1$

NN problem "solved" !!



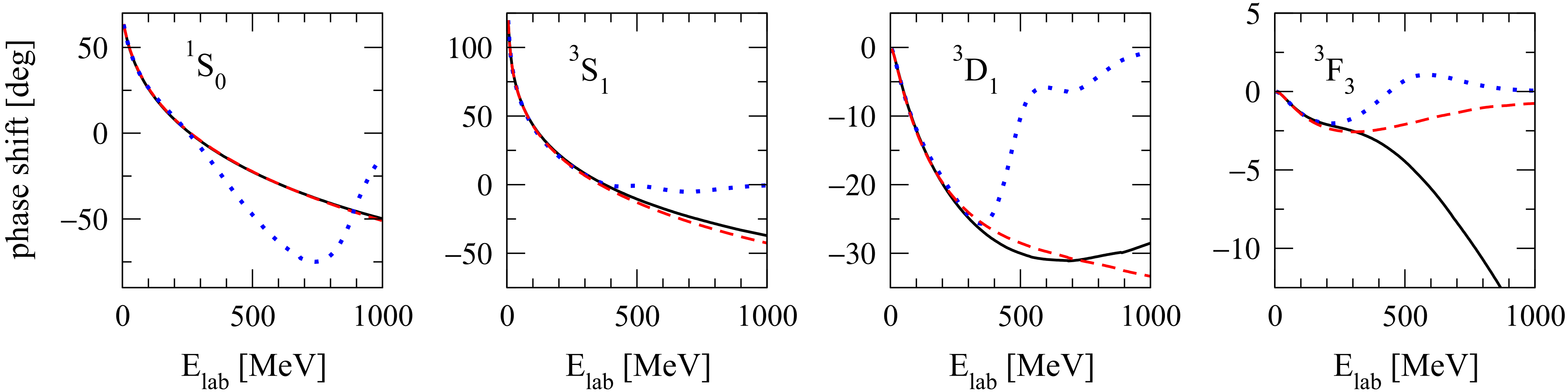




Many successes, but…

1) Difficult (impossible) to assign theoretical error
2) 3N forces (what are those??) not consistent with NN forces
3) No clear connection to QCD
4) Clear **model dependence**…

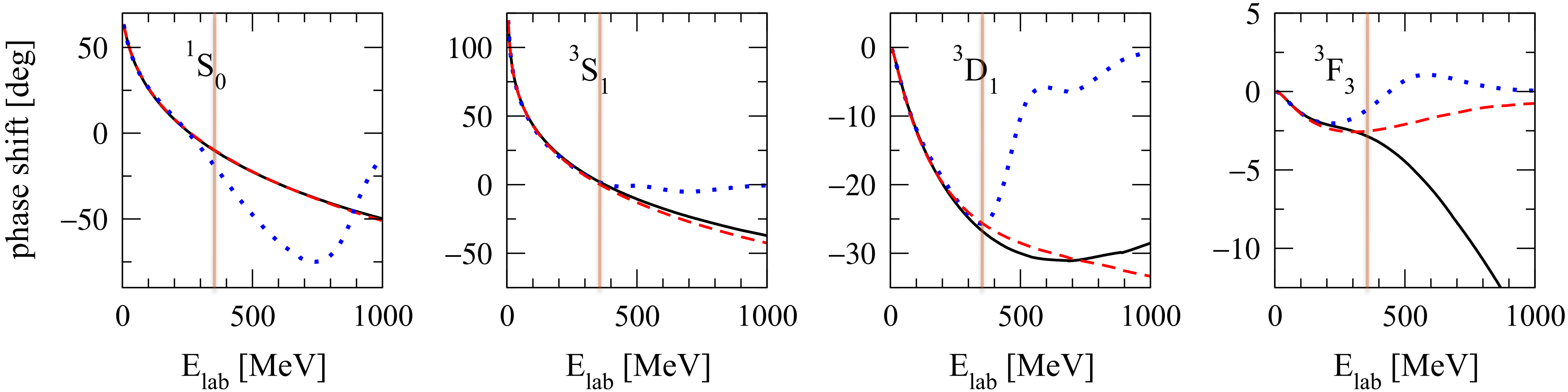
# Meson-Exchange Potentials and Phase Shifts

Further model dependence: scattering phase shifts of NN potentials

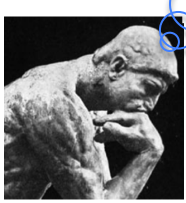


# Meson-Exchange Potentials and Phase Shifts

Further model dependence: scattering phase shifts of NN potentials



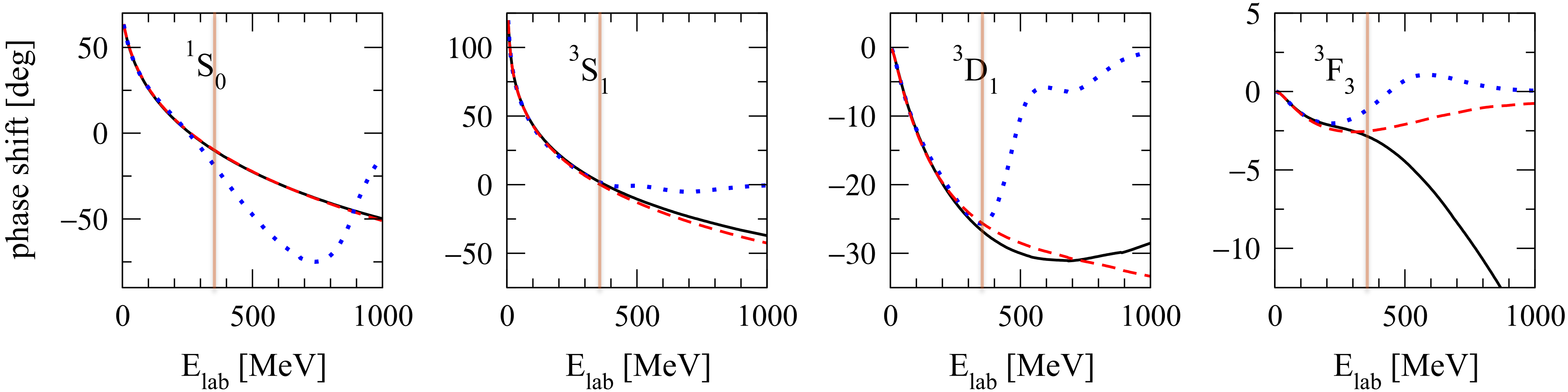

That's strange…
why do they only
agree to 350MeV?

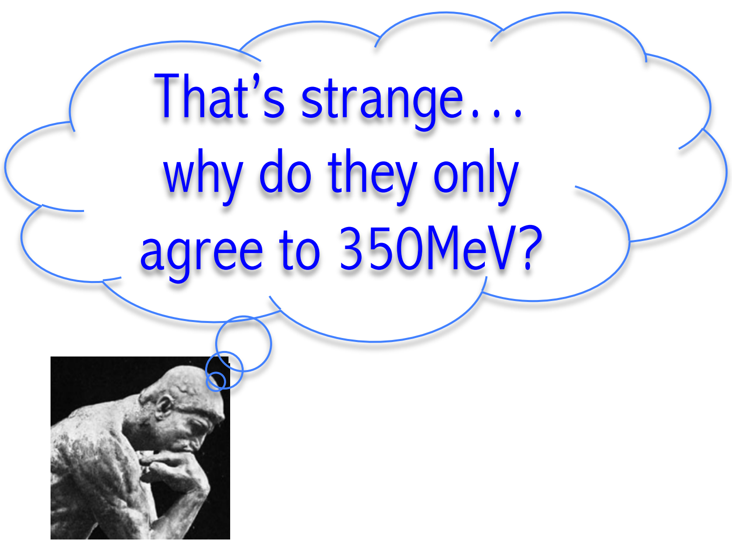


Remember, all have $\chi^2/\mathrm{dof} \approx 1$

# Meson-Exchange Potentials and Phase Shifts

Further model dependence: scattering phase shifts of NN potentials



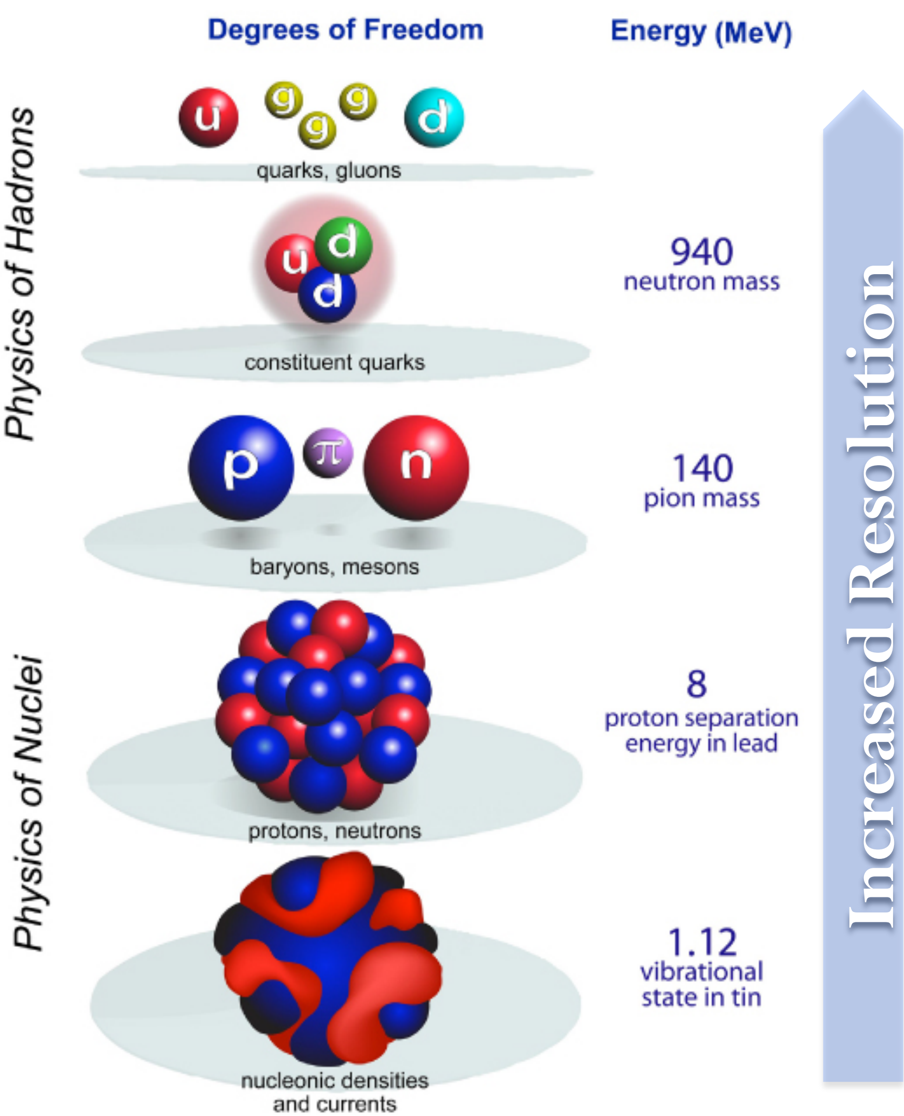

Agree well up to **pion-production threshold** ~350MeV

Most models don't fit phase shifts above this energy – **unconstrained**

# From QCD to Nuclear Interactions

**How do we determine interactions between nucleons?**

$$H(\Lambda) = T + V_{\mathrm{NN}}(\Lambda) + V_{\mathrm{3N}}(\Lambda) + V_{\mathrm{4N}}(\Lambda) + \cdots$$




**Old view:**

Multiple scales complicate life

No meaningful way to connect them

**Modern view:**

**Ratio** of scales – small parameters

Effective field theory at each scale connected by RG

Choose convenient resolution scale