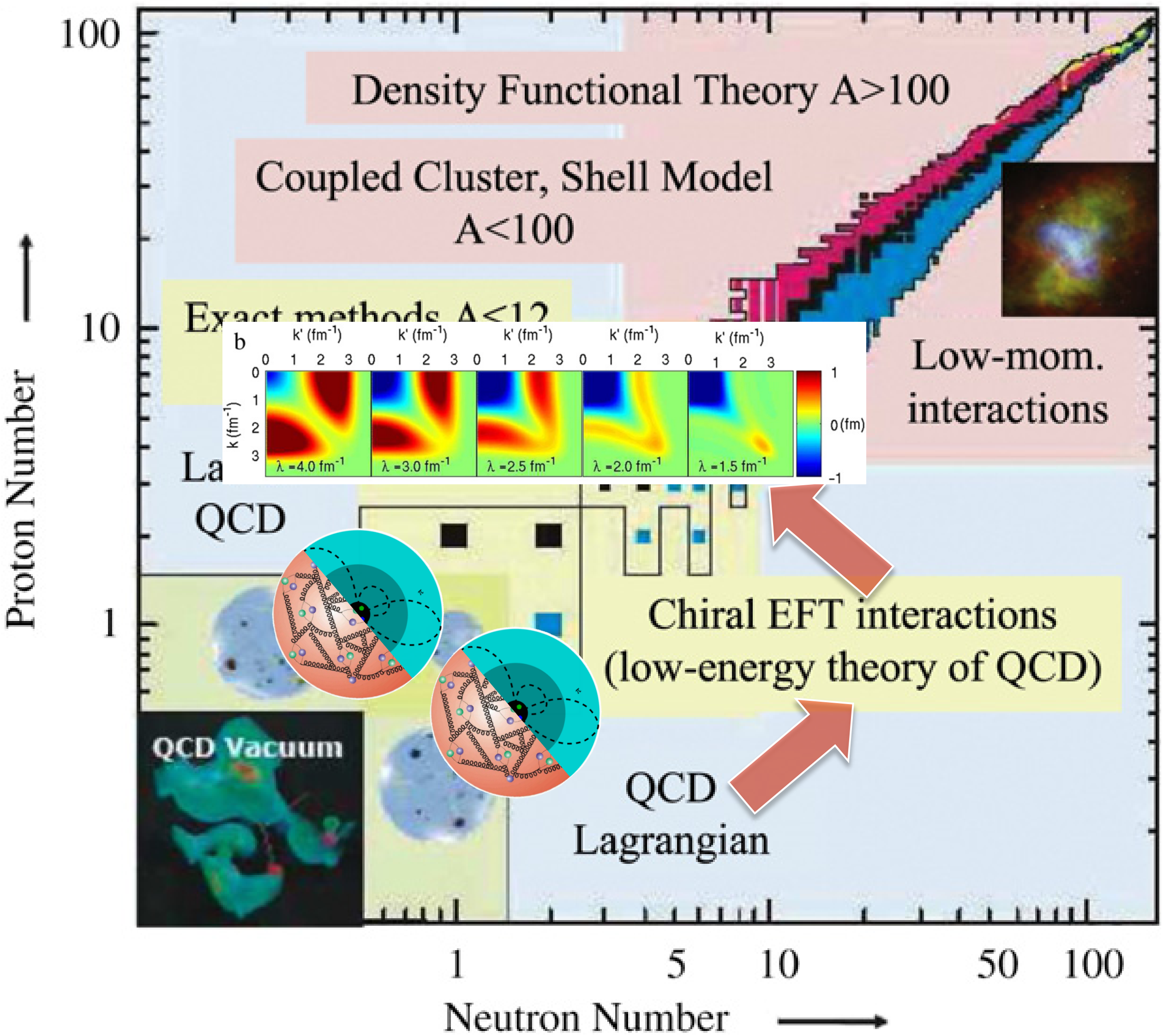
Part II: (S)RG and Low-Momentum Interactions

To understand the properties of complex nuclei from first principles



Renormalizing NN Interactions

Basic ideas of RG

Low-momentum interactions

Similarity RG interactions

Benefits of low cutoffs

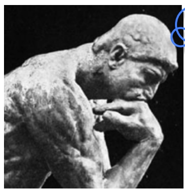
G-matrix renormalization

How will we approach this problem:

QCD → NN (3N) forces → Renormalize → "Solve" many-body problem → Predictions

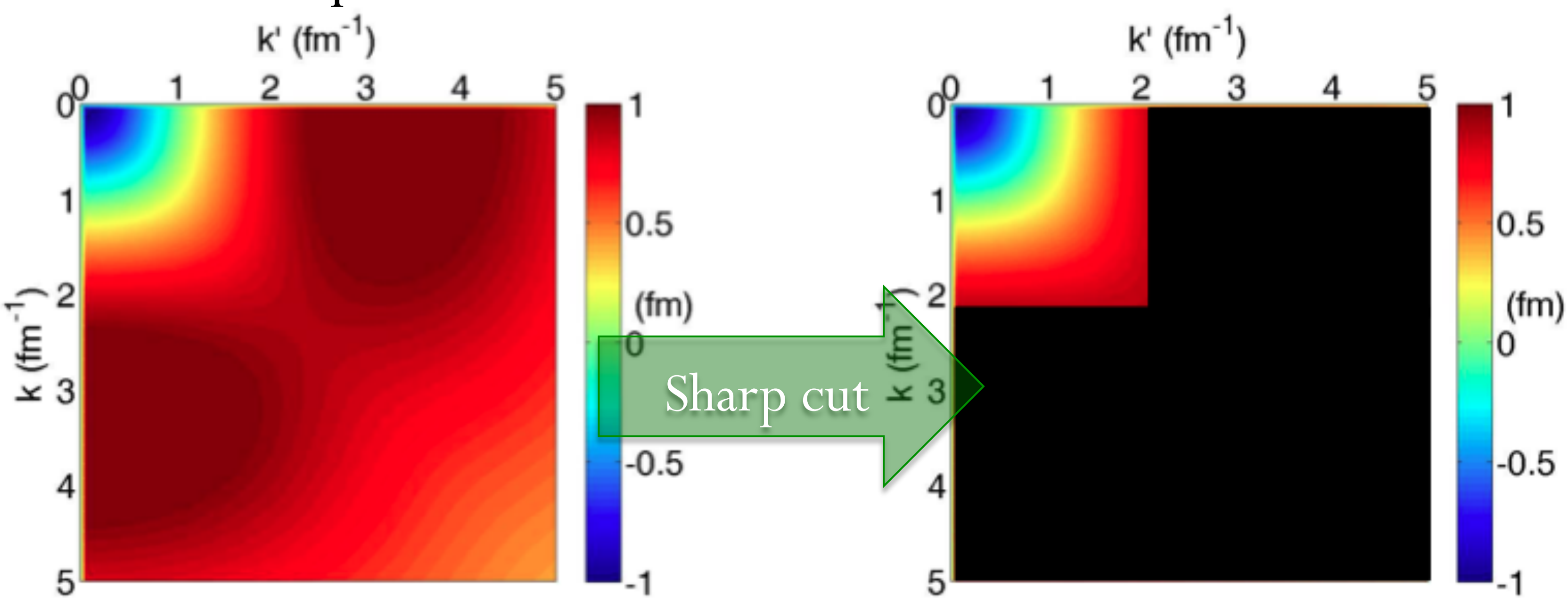
Renormalization of Meson-Exchange Potentials

Ok, high momentum is a pain. I wonder what would happen to low-energy observables…



Low-to-high momentum makes life difficult for low-energy nuclear theorists, so let's get rid of it

Can we just make a sharp cut and see if it works?

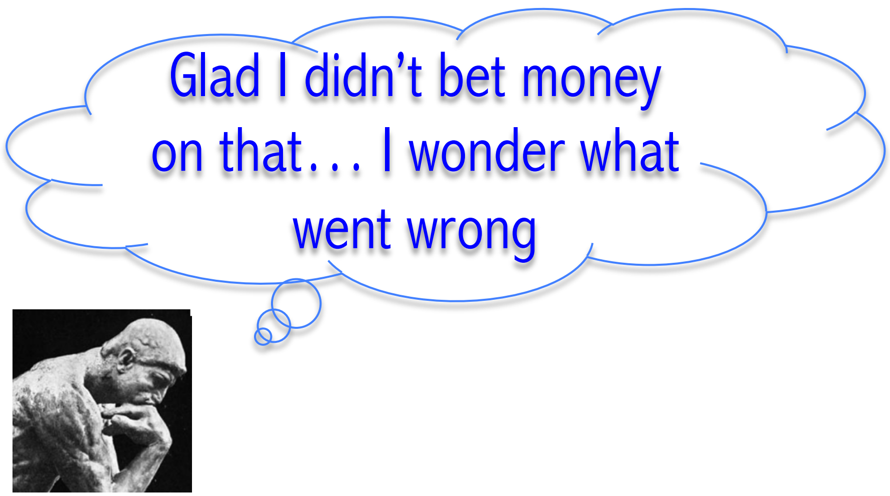


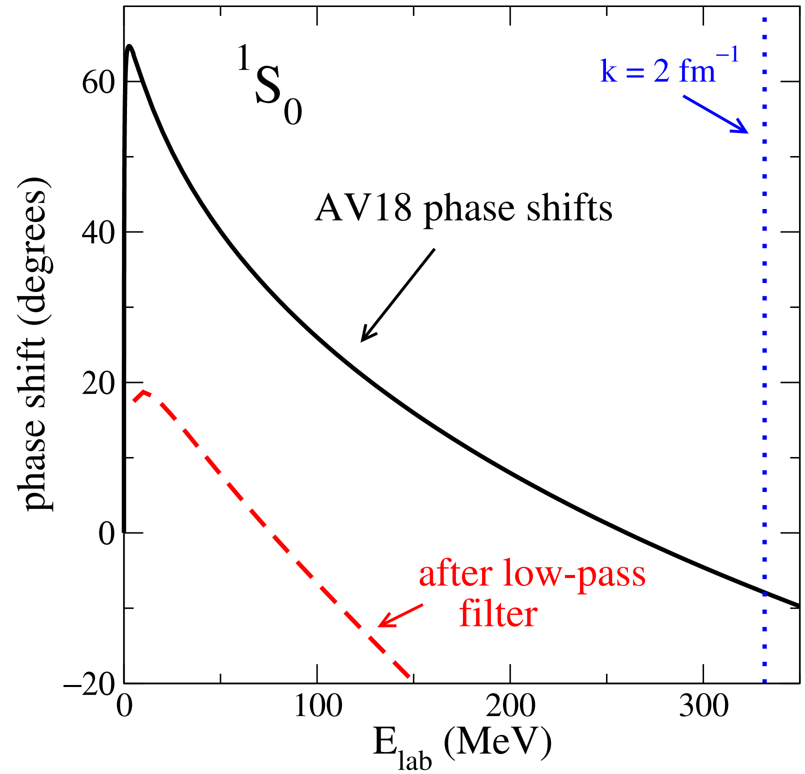
$$V_{\text{filter}}(k', k) \equiv 0; \;\; k, k' > 2.2\,\text{MeV}$$

Renormalization of Meson-Exchange Potentials

Can we just make a sharp cut?

Nope! Low-energy physics is not correct



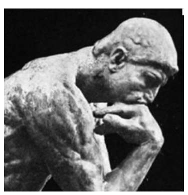


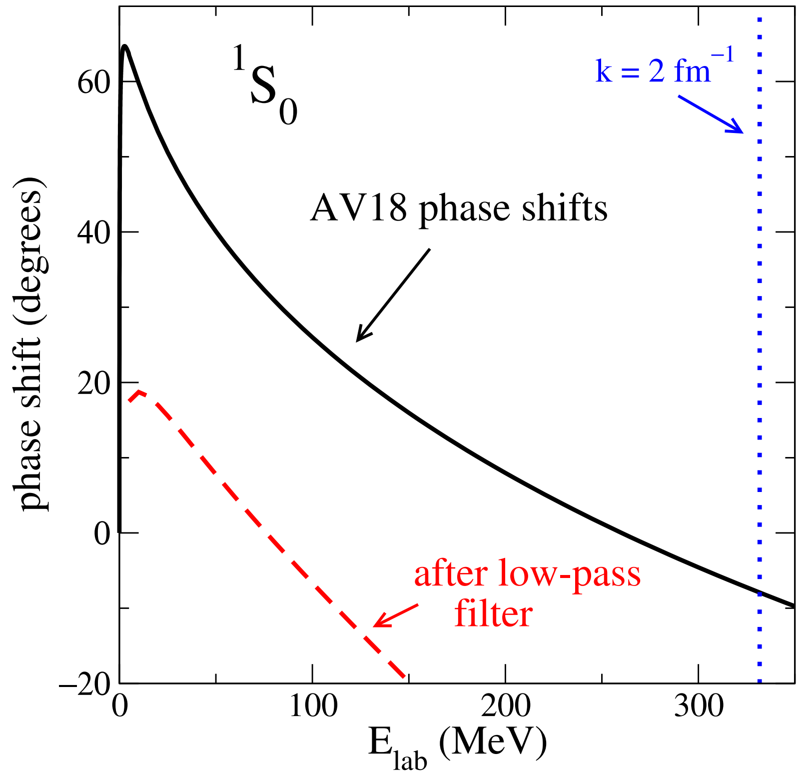
Renormalization of Meson-Exchange Potentials

Can we just make a sharp cut?

Nope! Low-energy physics is not correct

Glad I didn't bet money on that… I wonder what went wrong





Phase shifts involve couplings of low-to-high momenta

$$\langle k|V|k'\rangle + \sum_{q=0}^{\Lambda} \frac{\langle k|V|q\rangle\langle q|V|k'\rangle}{\epsilon_{k'} - \epsilon_q} + \sum_{q=\Lambda}^{\infty} \frac{\langle k|V|q\rangle\langle q|V|k'\rangle}{\epsilon_{k'} - \epsilon_q}$$

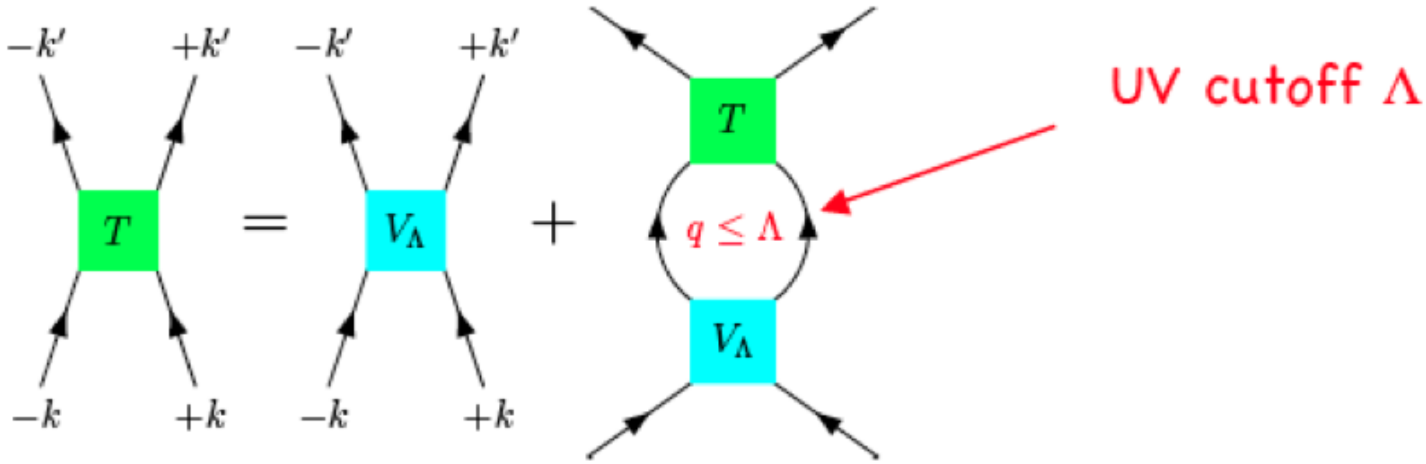
Lesson: Must ensure low-energy physics is preserved!

Renormalization of Meson-Exchange Potentials

To do properly, from T-matrix equation, define **low-momentum** equation:

$$T^{\alpha}(k,k') = V^{\alpha}_{\rm NN}(k,k') + \frac{2}{\pi}\int_0^{\infty} q^2 \mathrm{d}q \, \frac{V^{\alpha}_{\rm NN}(k,q)T^{\alpha}(q,k')}{k^2-q^2+i\varepsilon}$$

$$\rightarrow V^{\Lambda}_{{\rm low}\,k}(k,k') + \frac{2}{\pi}\int_0^{\Lambda} q^2 \mathrm{d}q \, \frac{V^{\Lambda}_{{\rm low}\,k}(k,q)T(q,k')}{k^2-q^2+i\varepsilon}$$



Lower UV cutoff, but preserve low-energy physics!

Renormalization of Meson-Exchange Potentials

To do properly, from T-matrix equation, define **low-momentum** equation:

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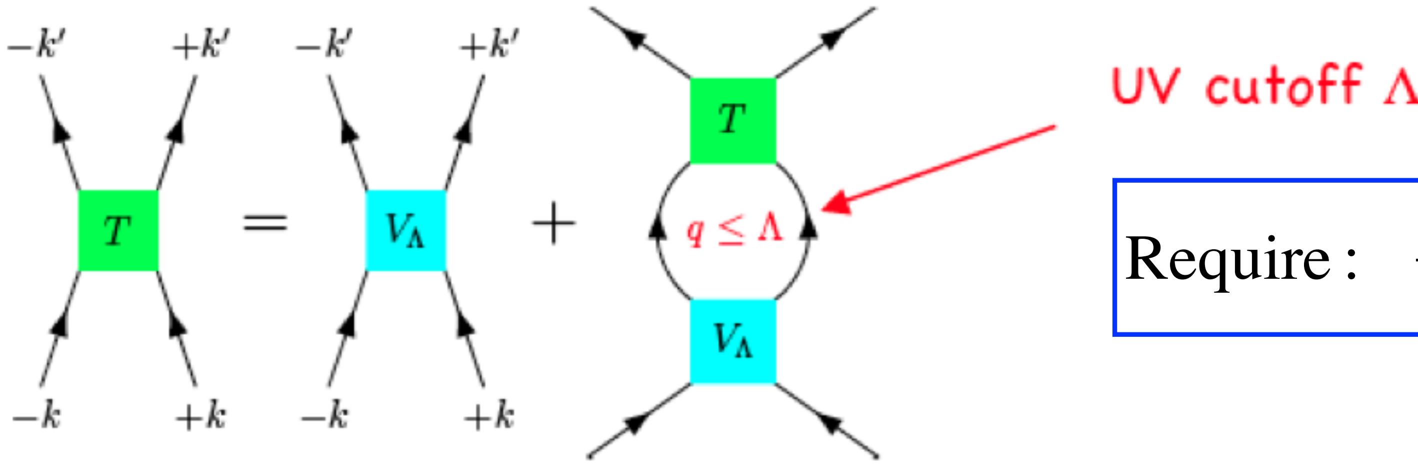
$$\rightarrow V_{{\rm low}\,k}^{\Lambda}(k,k') + \frac{2}{\pi}\int_0^{\Lambda} q^2 dq\, \frac{V_{{\rm low}\,k}^{\Lambda}(k,q)T(q,k')}{k^2-q^2+i\varepsilon}$$



$$\boxed{\text{Require}: \quad \frac{d}{d\Lambda} T = 0}$$

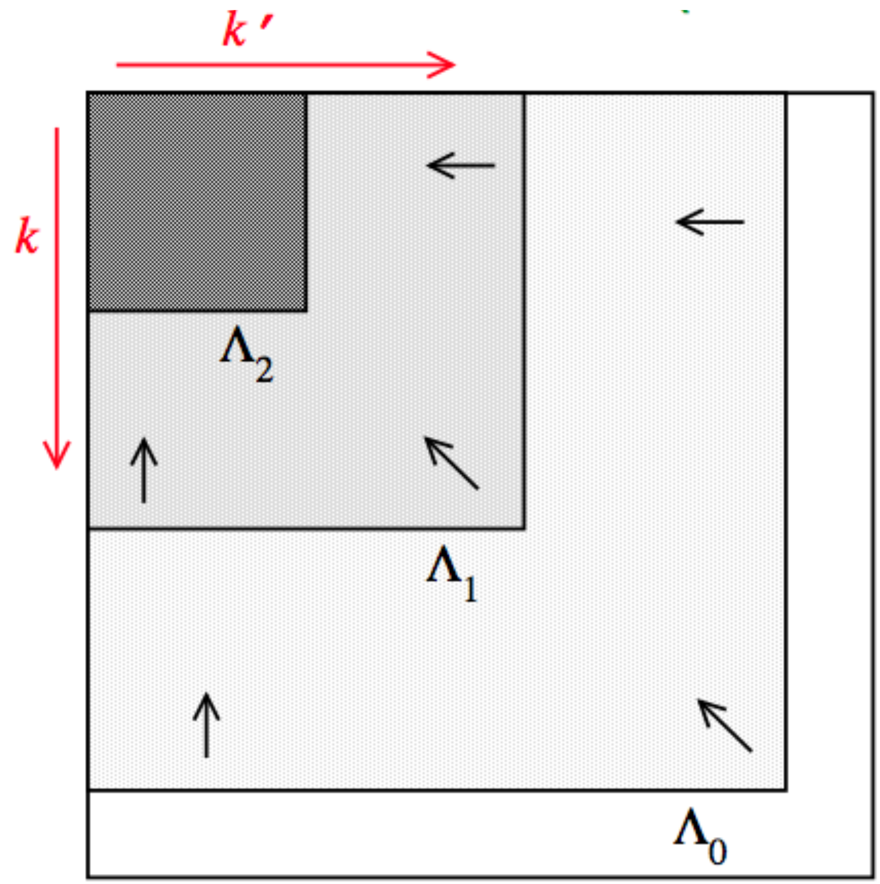
Lower UV cutoff, but preserve low-energy physics!

Leads to **renormalization group equation** for low-momentum interactions

$$\frac{d}{d\Lambda} V_{{\rm low}\,k}^{\Lambda}(k',k) = \frac{2}{\pi}\frac{V_{{\rm low}\,k}^{\Lambda}(k',\Lambda)T^{\Lambda}(\Lambda,k)}{1-(k/\Lambda)^2}$$

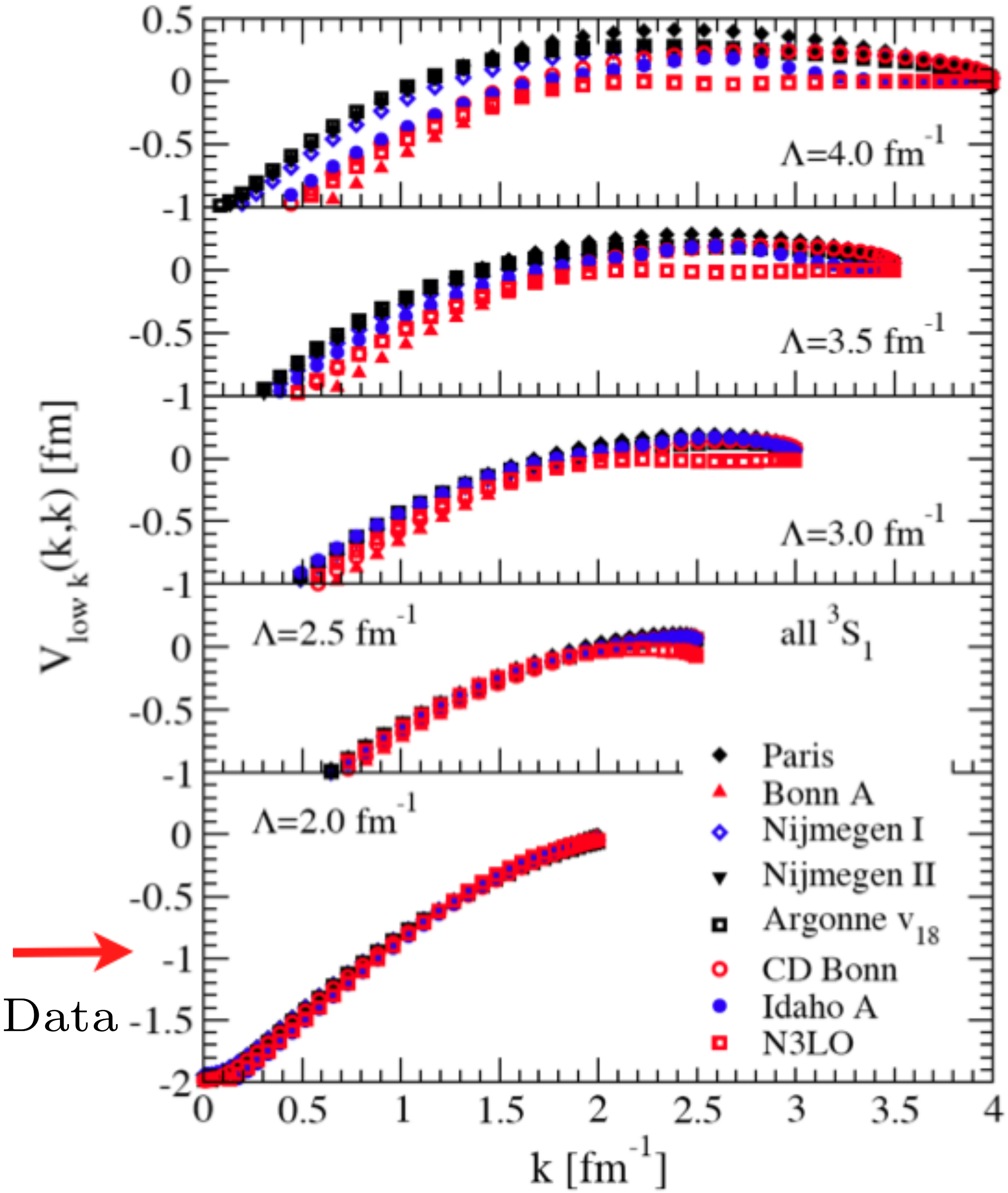
Renormalization of Meson-Exchange Potentials

Run cutoff to lower values – **decouples** high-momentum modes



Start from some initial V_{NN}
at high cutoff Λ_0

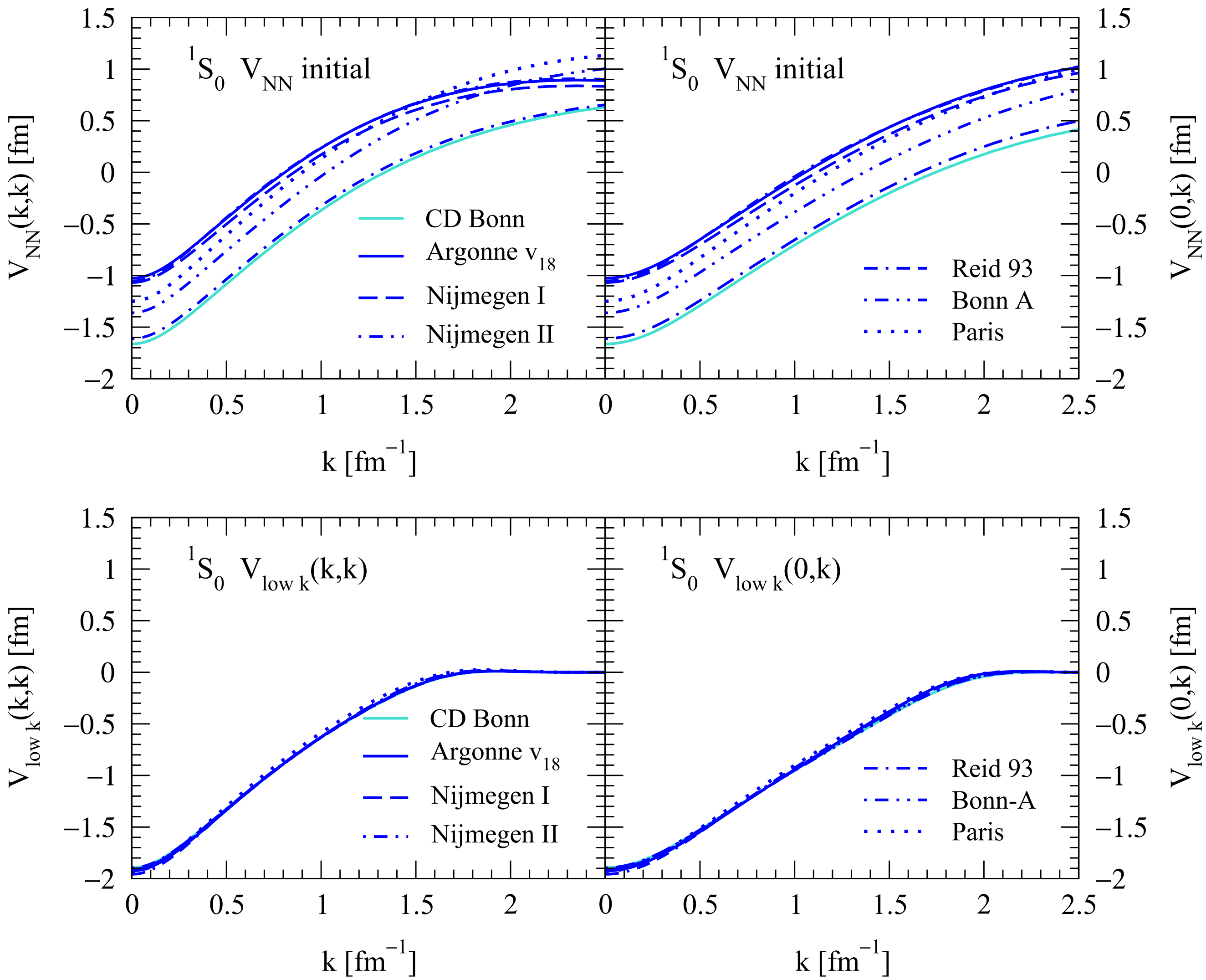
$\Lambda \approx \Lambda_{\mathrm{Data}}$



"Universality" at low momentum

Renormalization of Meson-Exchange Potentials

Diagonal **Off-diagonal**



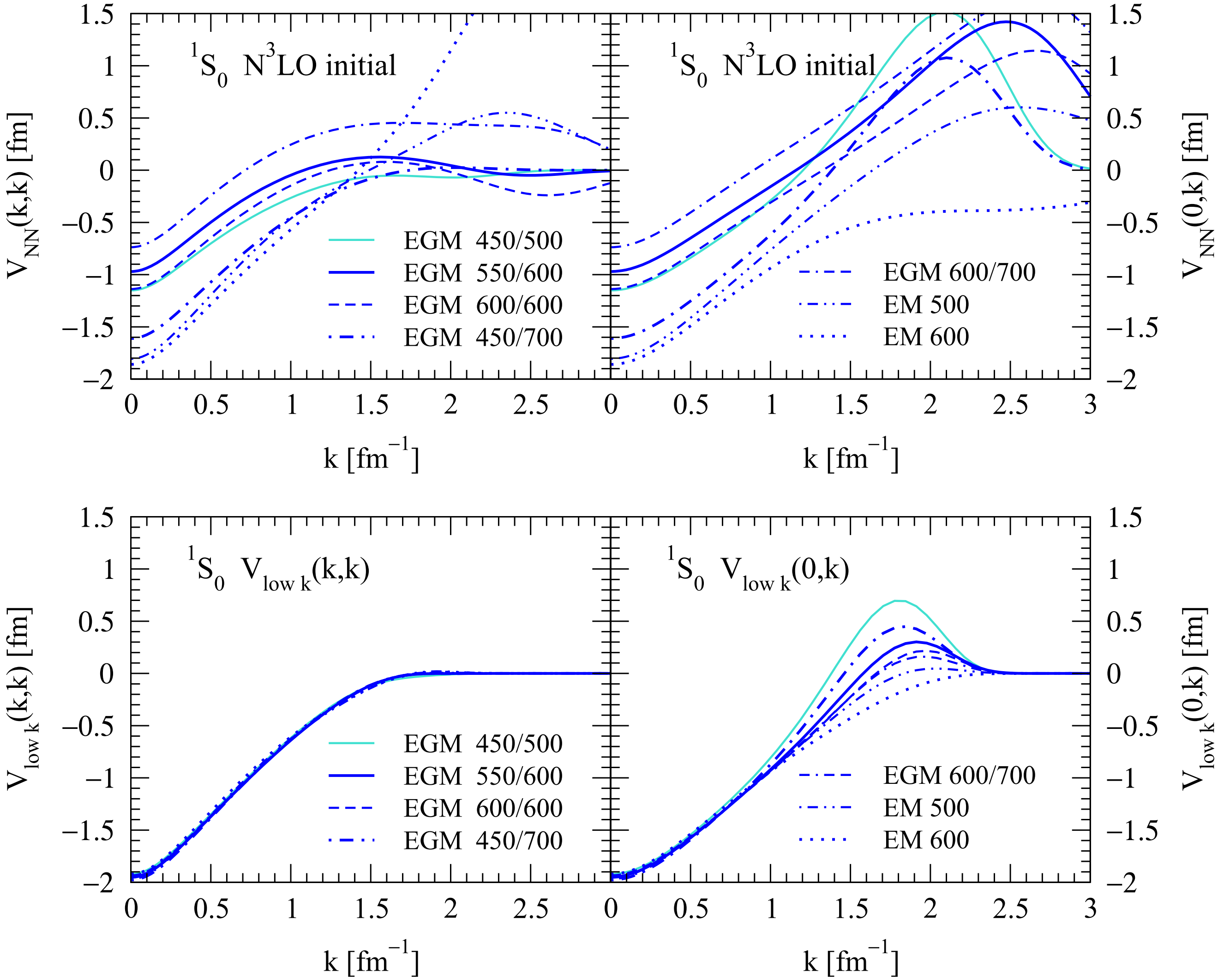
These are all our favorite OBE NN potentials…

These are all our favorite OBE NN potentials… **at low momentum**

Universal collapse in both diagonal/off-diagonal components, most partial waves

Renormalization of Chiral EFT Potentials

Diagonal **Off-diagonal**

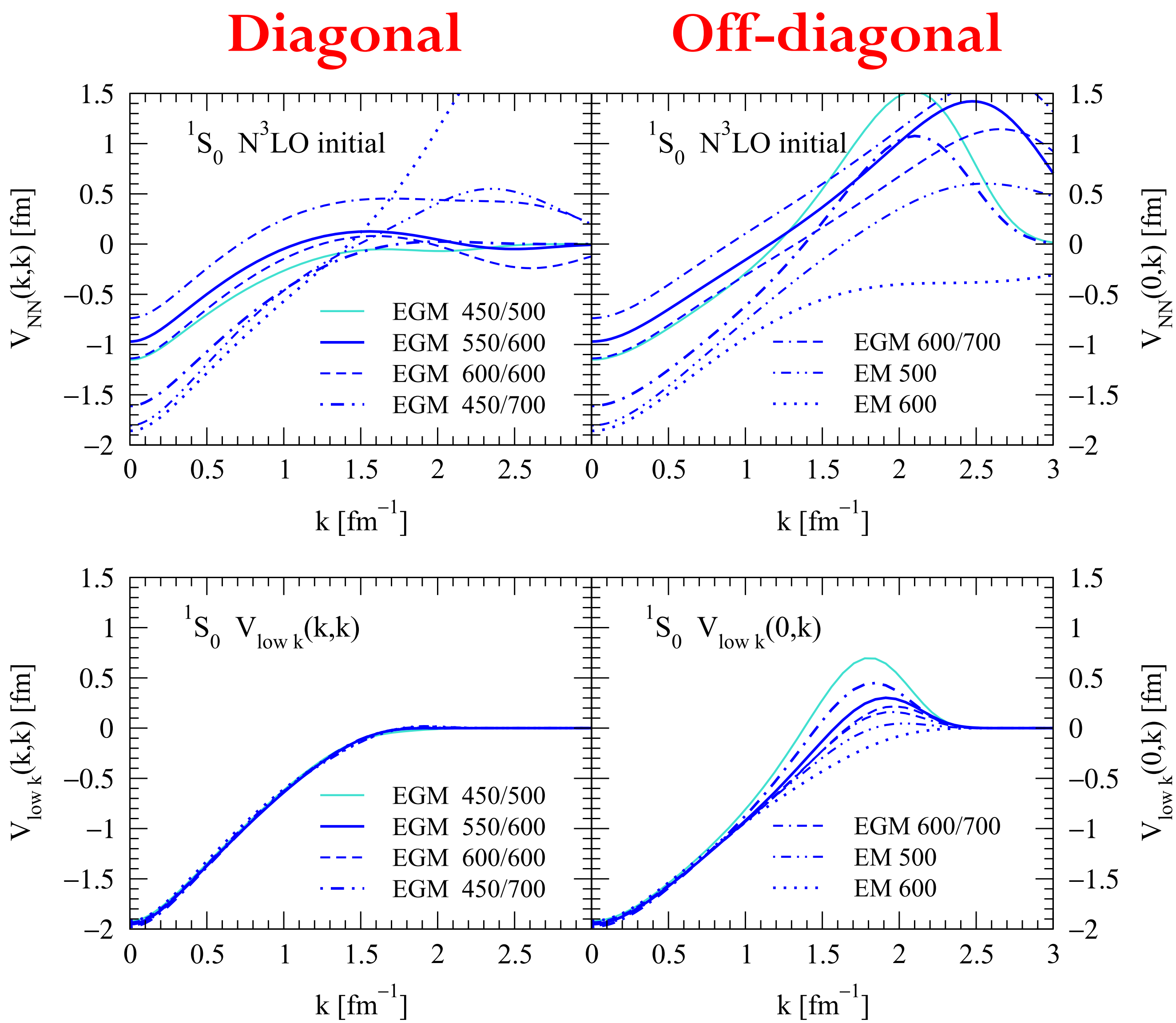


These are all our favorite Chiral EFT NN potentials…

These are all our favorite Chiral EFT NN potentials… **at low momentum**

Differences remain in off-diagonal matrix elements. Why?

Renormalization of Chiral EFT Potentials



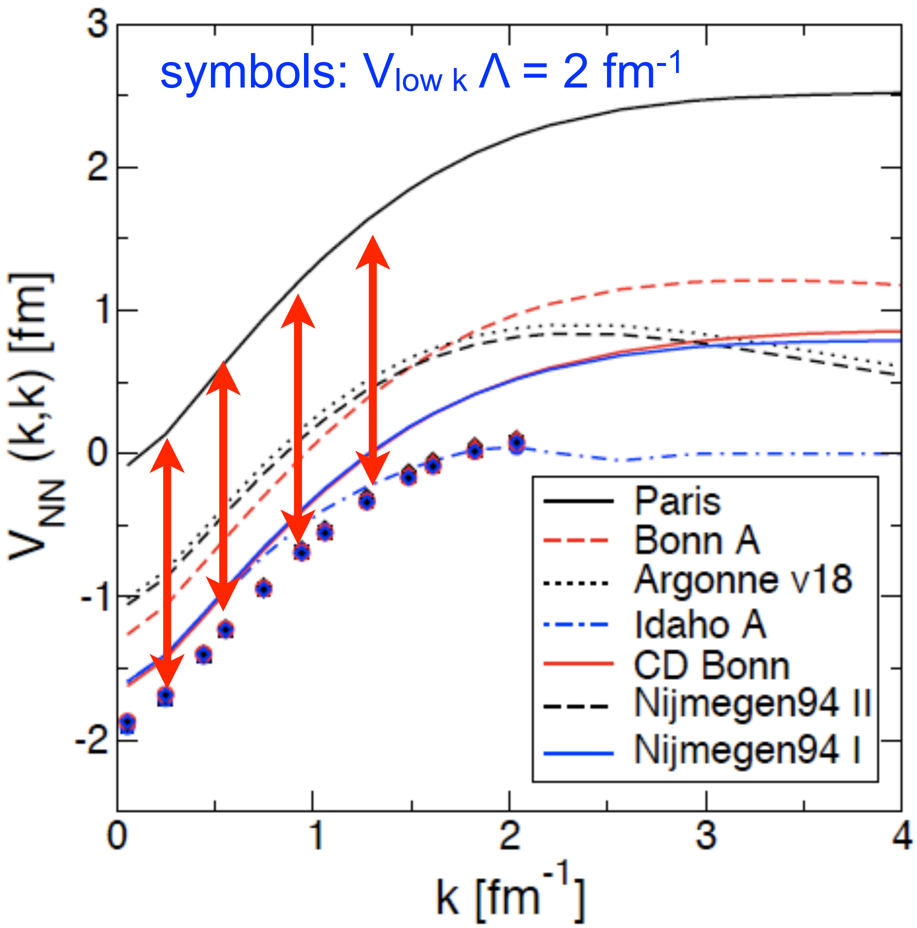
These are all our favorite Chiral EFT NN potentials…

These are all our favorite Chiral EFT NN potentials… **at low momentum**

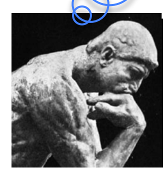
Differences remain in off-diagonal matrix elements

Sensitive to agreement for phase shifts (not all fit perfectly)

Renormalization of NN Potentials



Why is it mostly a shift?

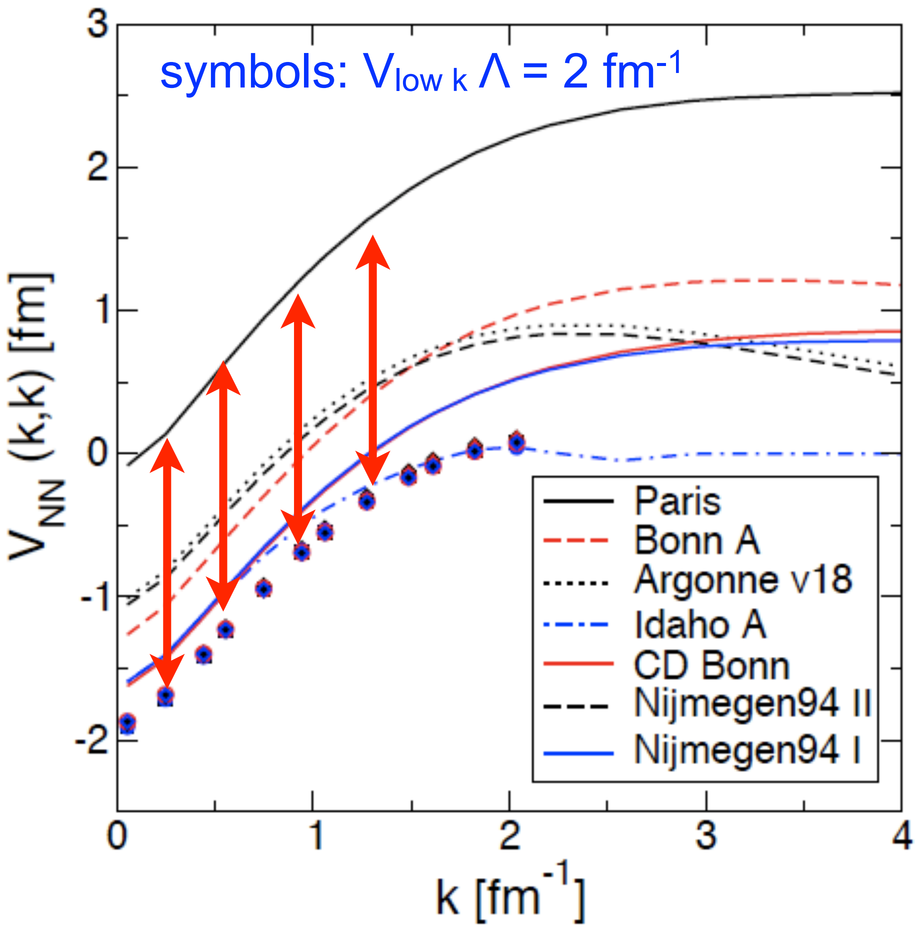


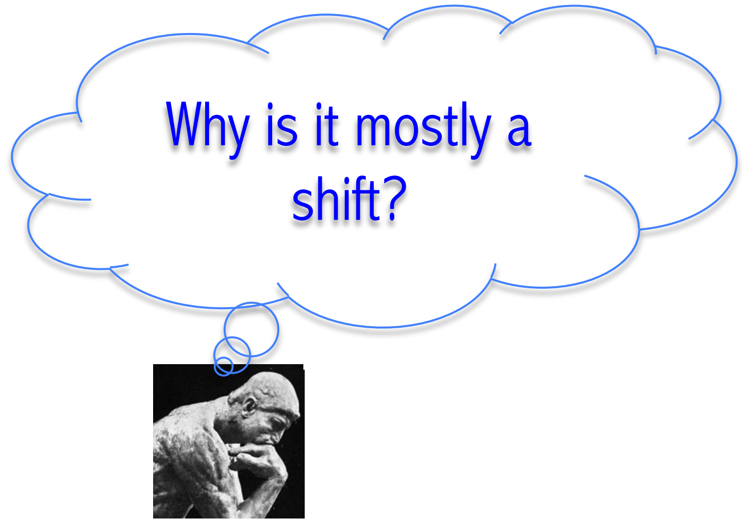
$$V_{\text{eff}} = V_{\text{L}} + \delta V_{\text{c.t.}}(\Lambda)$$

Overall effect of evolving to low momentum

Main effect is shift in momentum space

Renormalization of NN Potentials





$$V_{\text{eff}} = V_{\text{L}} + \delta V_{\text{c.t.}}(\Lambda)$$

Overall effect of evolving to low momentum

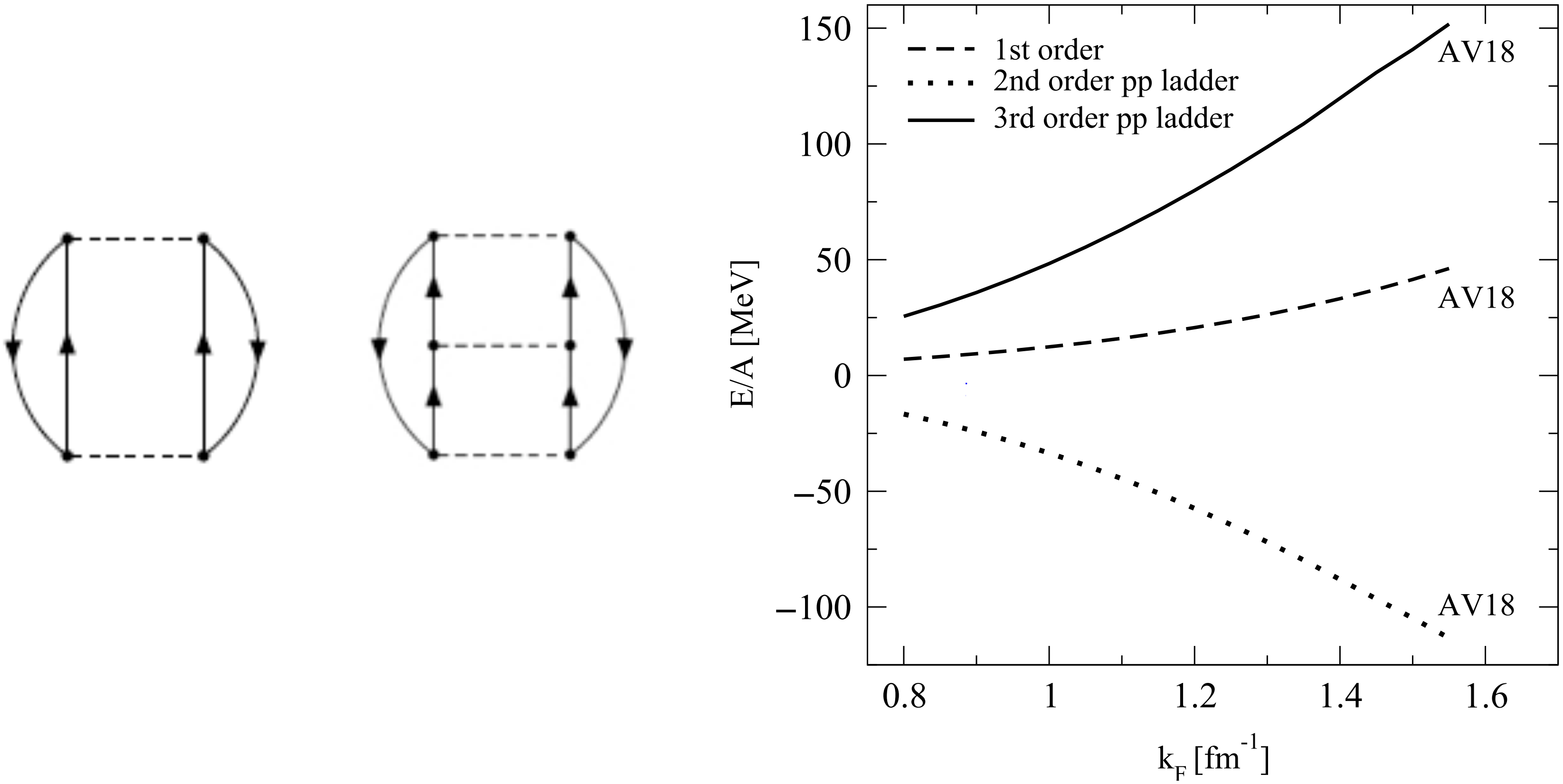
Main effect is shift in momentum space – delta function

Removes hard core (unconstrained short-range physics)!

Improvements in Perturbation Theory

Explore improvements in symmetric infinite matter calculations

Order by order in **many-body perturbation theory (MBPT)**



No clear convergence with increasing order in bare potential

Improvements in Perturbation Theory

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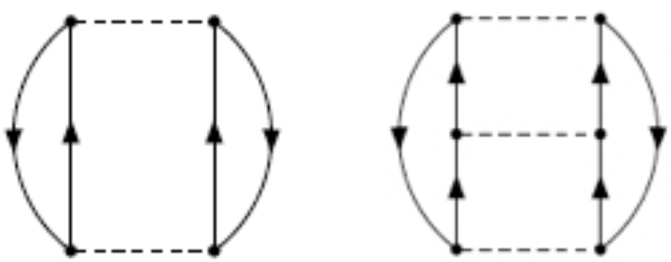

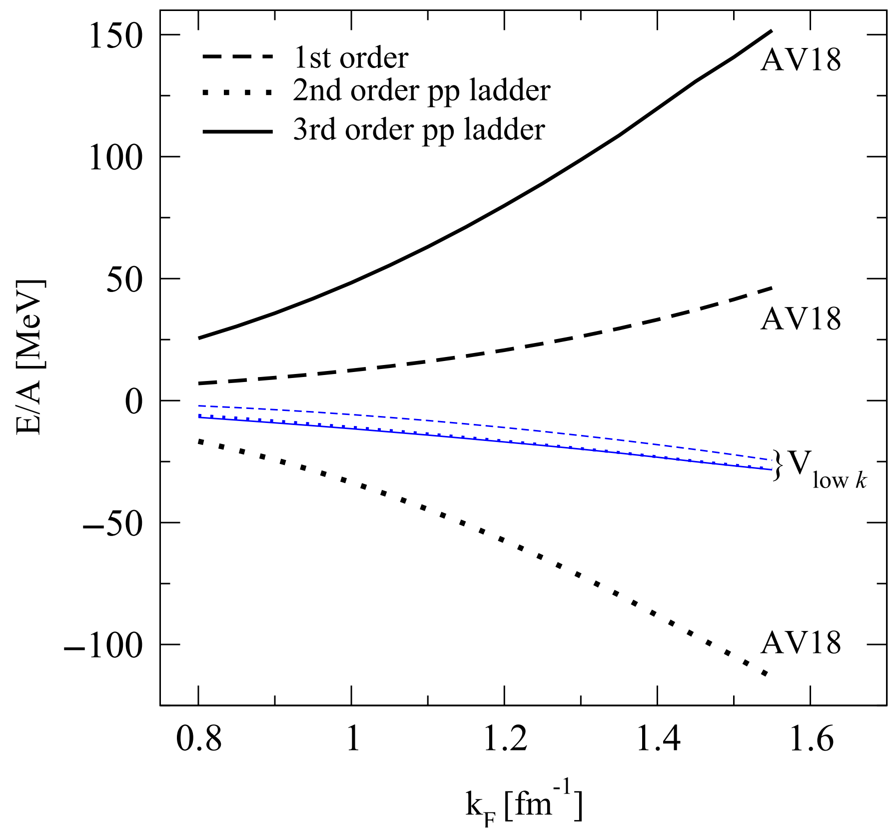

No clear convergence with increasing order in bare potential

Significant improvement with low-momentum interactions!

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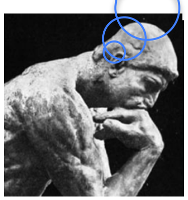
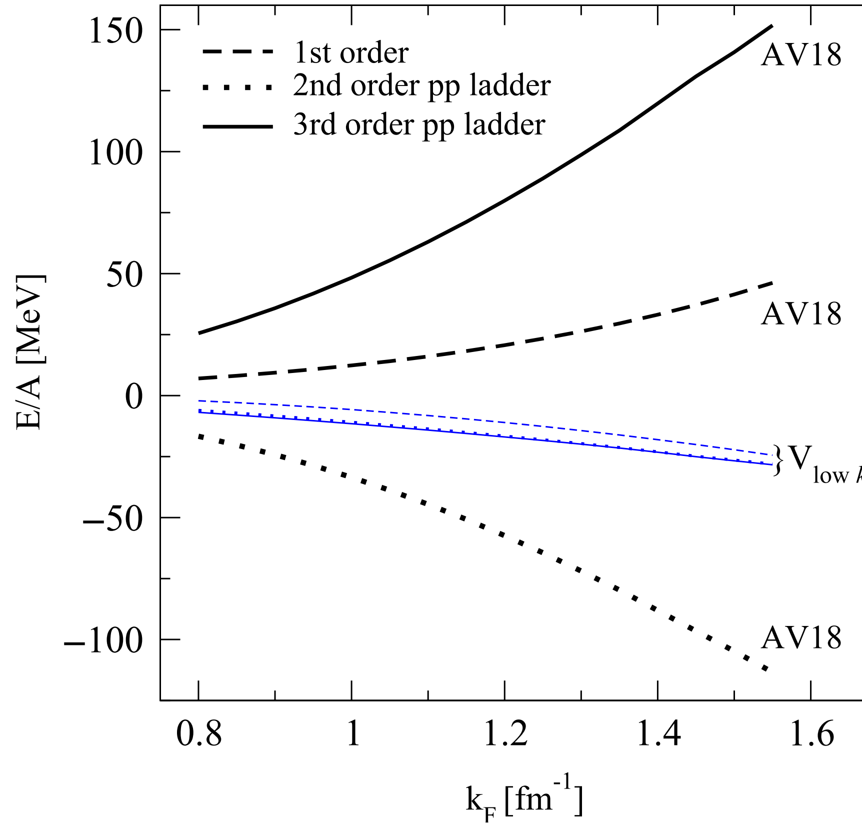
Ok, the interactions look perturbative, but something is wrong here…





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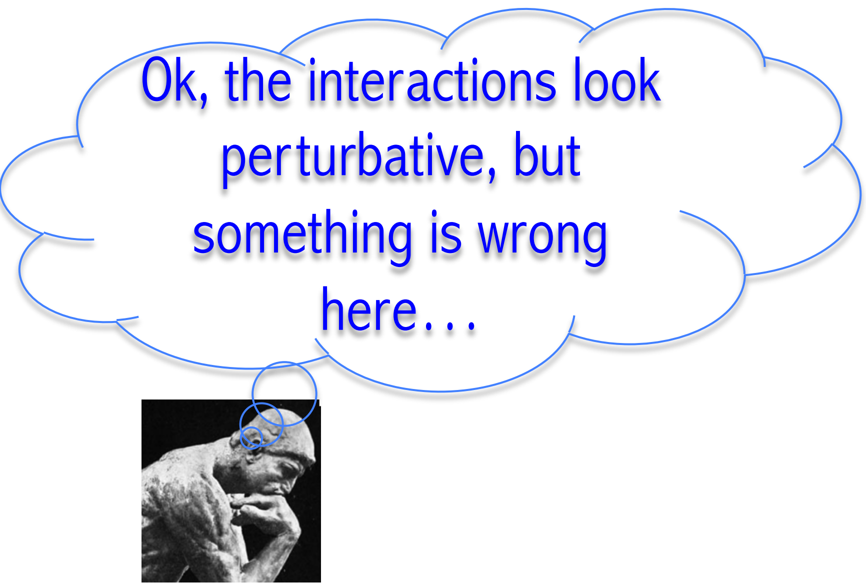
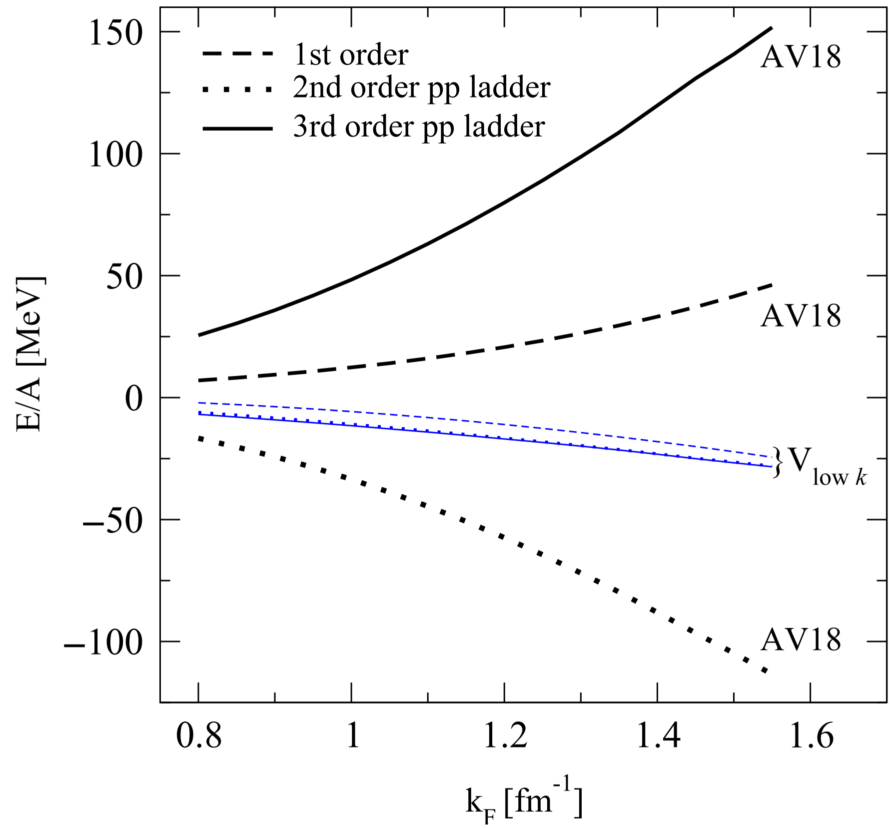




No clear convergence with increasing order in bare potential

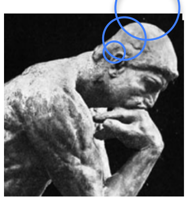
Significant improvement with low-momentum interactions!

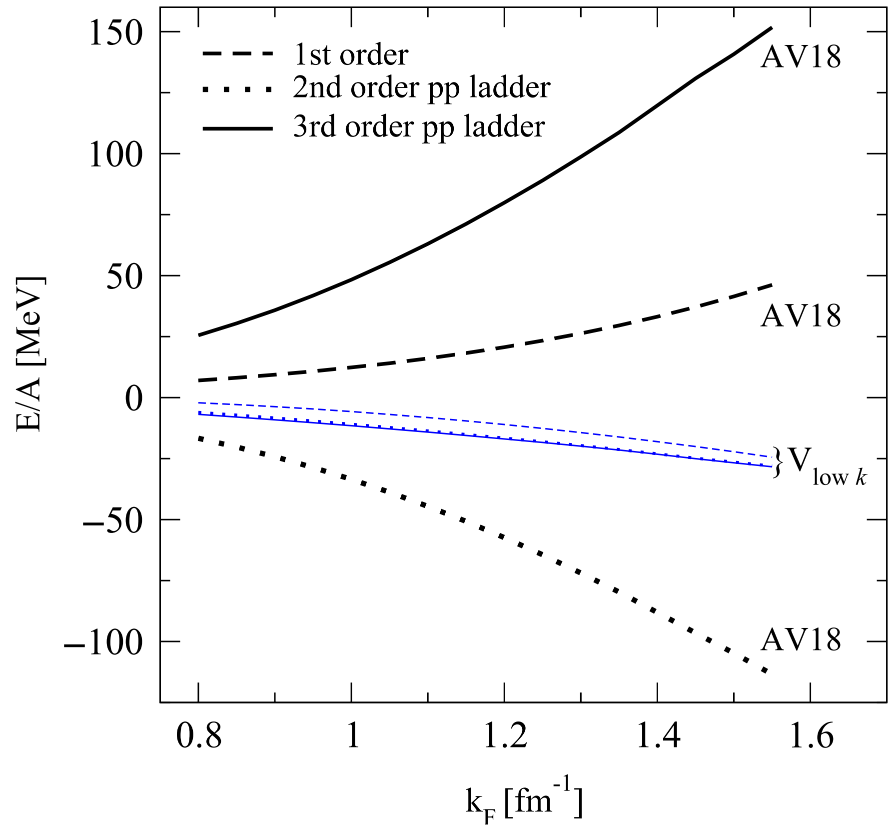
Does not saturate – what might be missing?

Improvements in Perturbation Theory

$$H(\Lambda) = T + V_{\mathrm{NN}}(\Lambda) + V_{3\mathrm{N}}(\Lambda) + V_{4\mathrm{N}}(\Lambda) + \cdots$$

Ok, the interactions look perturbative, but something is wrong here…





No clear convergence with increasing order in bare potential

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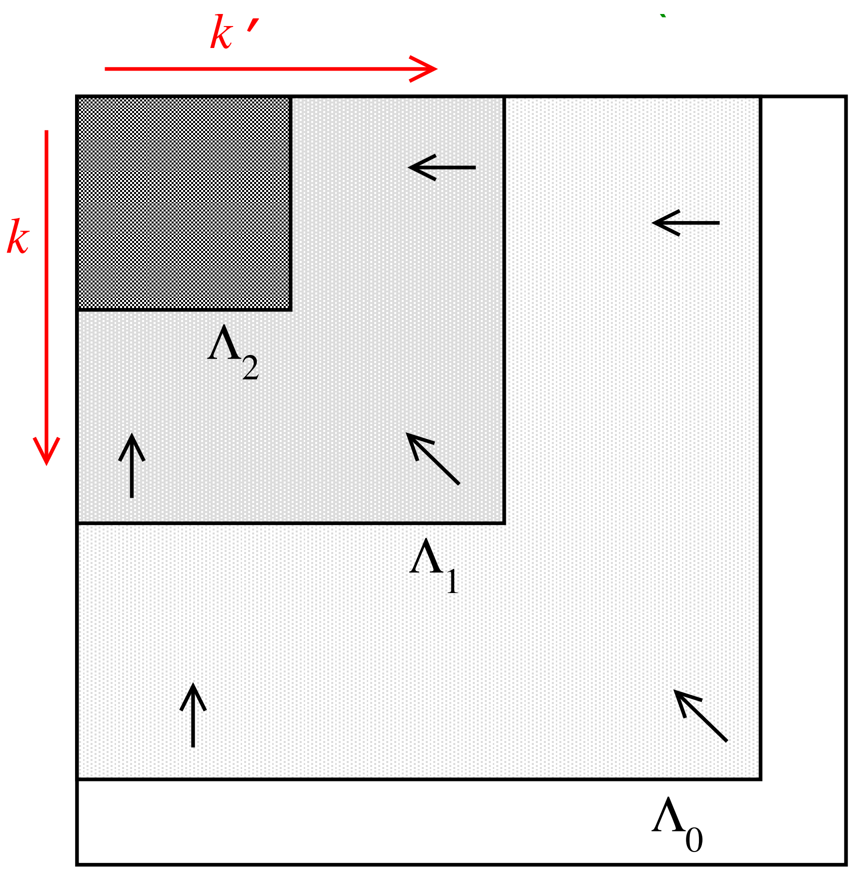
Does not saturate – what might be missing?

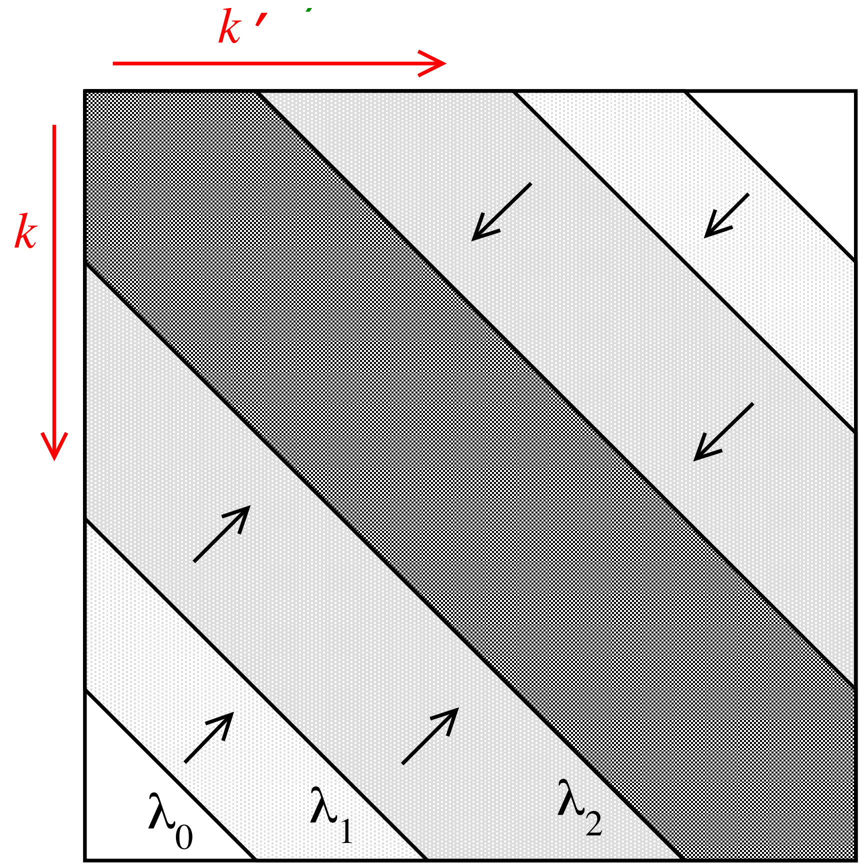
Similarity Renormalization Group

Wegner, Glazek/Wilson (1990s)

Complementary method to decouple low from high momenta



Decouples high-momentum



Similarity Renormalization Group

Drives Hamiltonian to band-diagonal

Similarity Renormalization Group

Wegner, Glazek/Wilson (1990s)

Apply a continuous unitary transformation, parameterized by s:

$$H = T + V \to H(s) = U(s) H U^{\dagger}(s) \equiv T + V(s)$$

where differentiating (exercise) yields:

$$\frac{\mathrm{d}H(s)}{\mathrm{d}s} = [\eta(s), H(s)] \quad \text{where} \quad \eta(s) \equiv \frac{\mathrm{d}U(s)}{\mathrm{d}s} U^{\dagger}(s)$$

Never explicitly construct unitary transformation

Instead **choose generator to obtain desired behavior**:

$$\eta(s) = [G(s), H(s)]$$

Many options, e.g.,

$$\eta(s) = [T, H(s)]$$ Drives H(s) to band-diagonal form

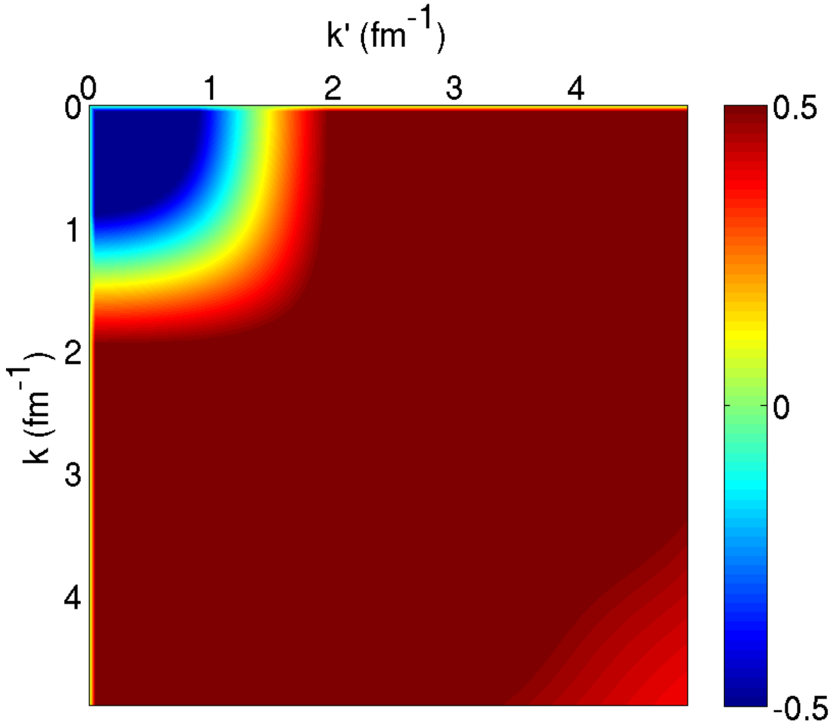
Illustration of SRG Flow

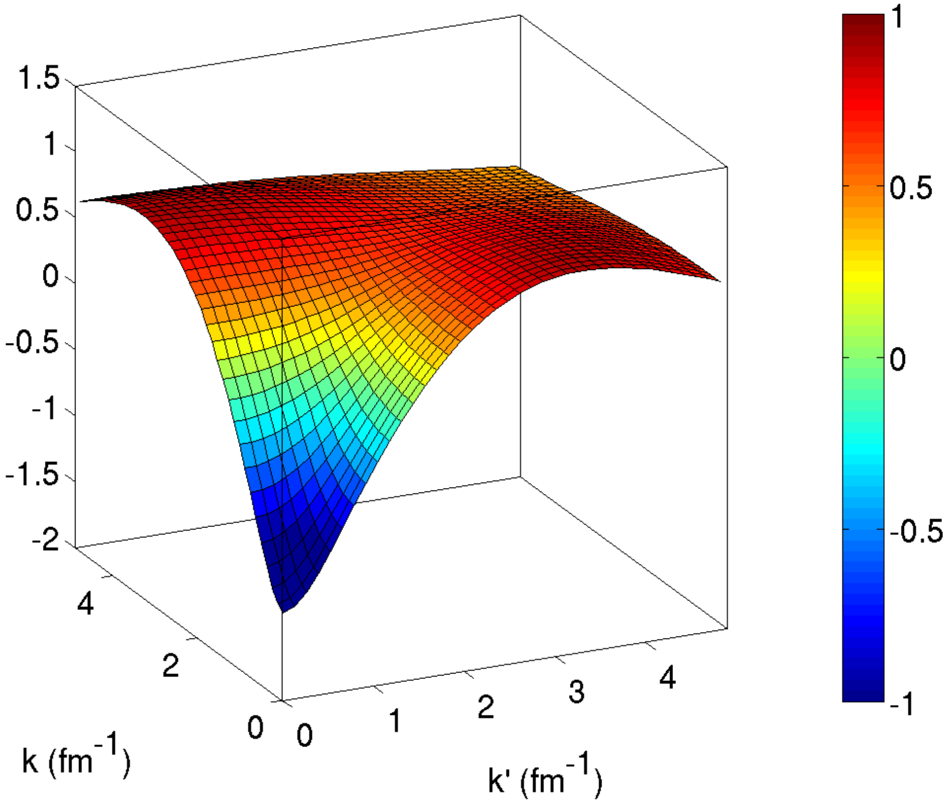
Drive H to band-diagonal form with kinetic-energy generator:

$$\eta(s) = [T, H(s)]$$

With alternate definition of flow parameter: $\lambda^2 = \dfrac{1}{\sqrt{s}}$

Argonne V_{18} $^1S_0$





$\lambda = 8.0\,\mathrm{fm}^{-1}$

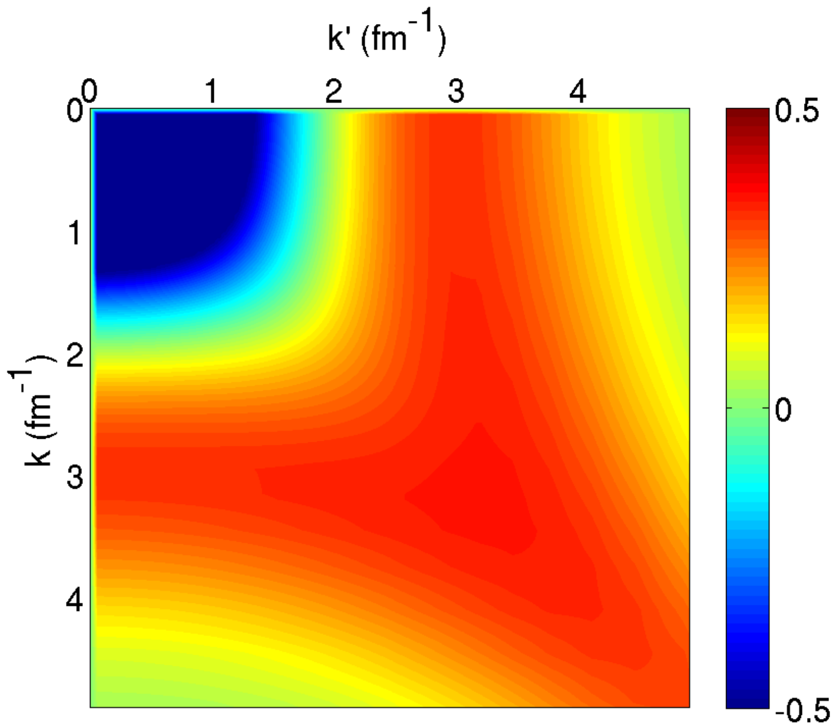
Illustration of SRG Flow

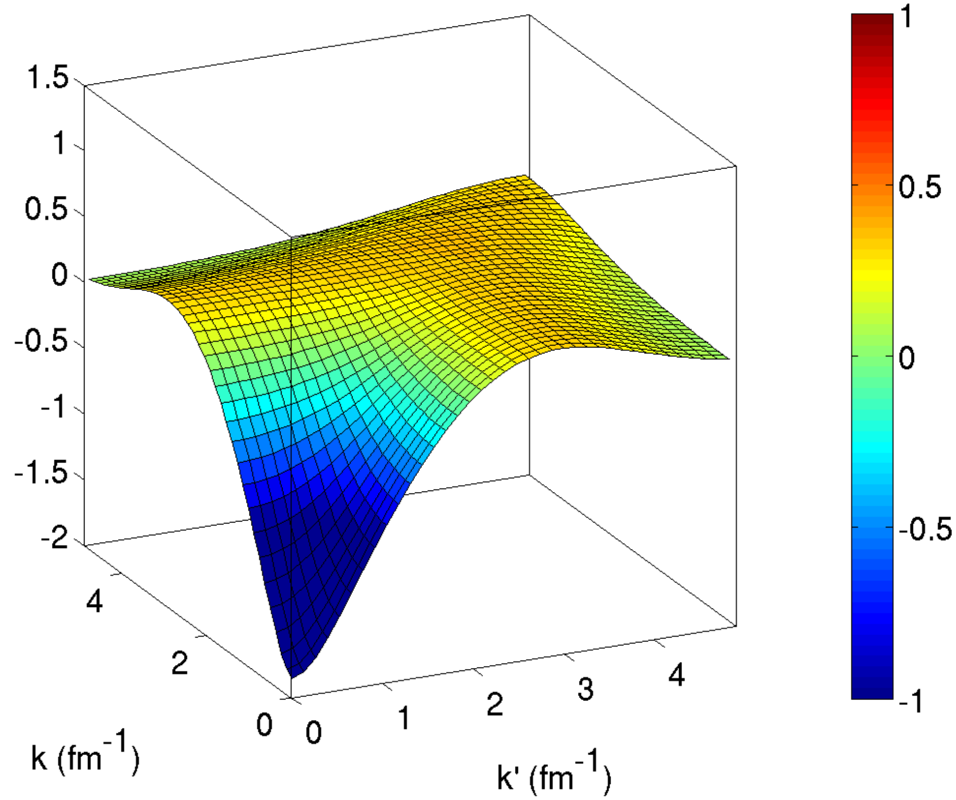
Drive H to band-diagonal form with standard choice:

$$\eta(s) = [T, H(s)]$$

With alternate definition of flow parameter: $\lambda^2 = \dfrac{1}{\sqrt{s}}$

Argonne V_{18} $\;{}^1S_0$





$\lambda = 4.0\,\mathrm{fm}^{-1}$

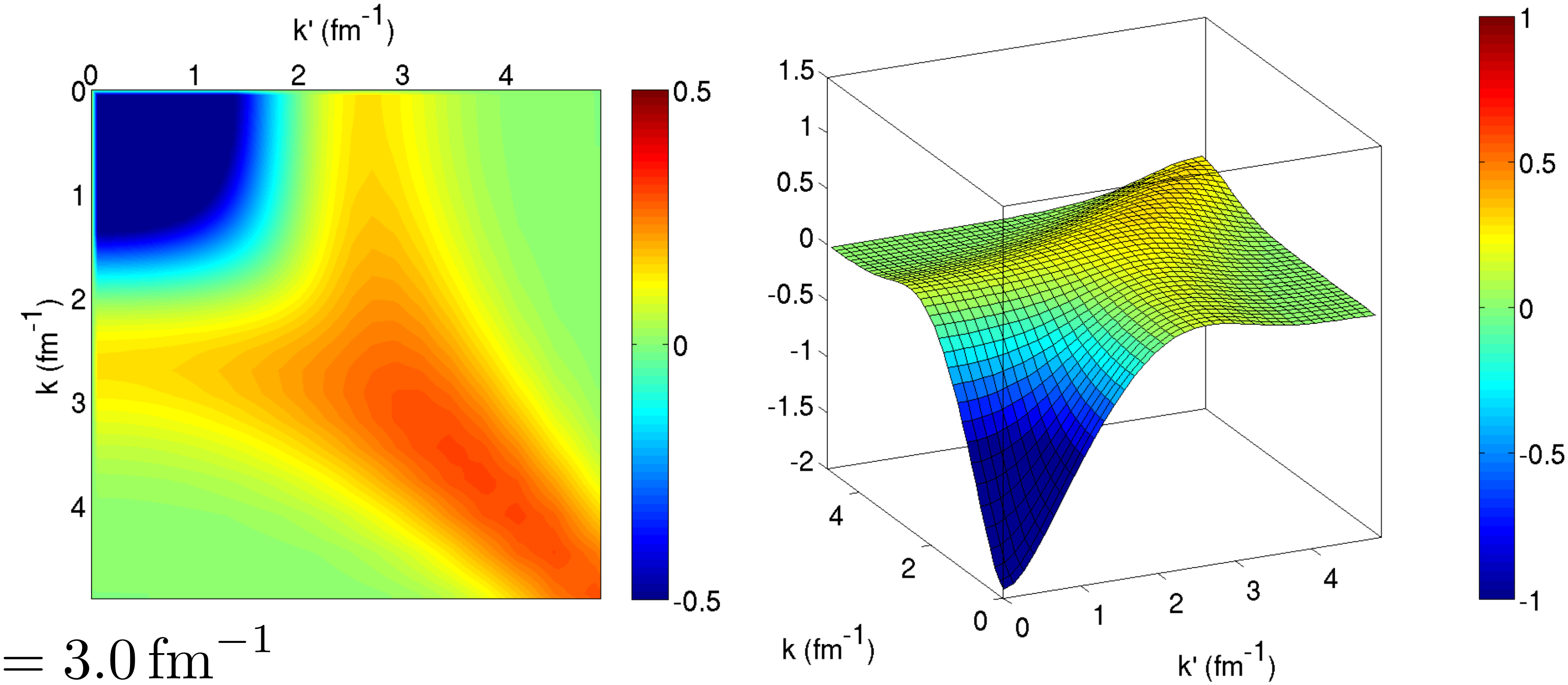
Illustration of SRG Flow

Drive H to band-diagonal form with standard choice:

$$\eta(s) = [T, H(s)]$$

With alternate definition of flow parameter: $\lambda^2 = \frac{1}{\sqrt{s}}$

Argonne V_{18} $^1S_0$



$\lambda = 3.0\,\mathrm{fm}^{-1}$

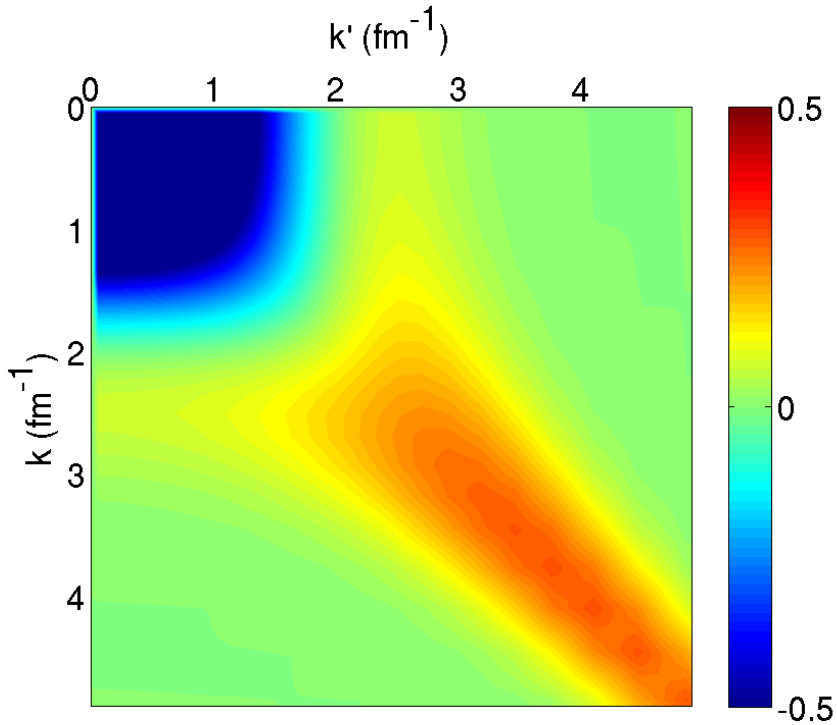
Illustration of SRG Flow

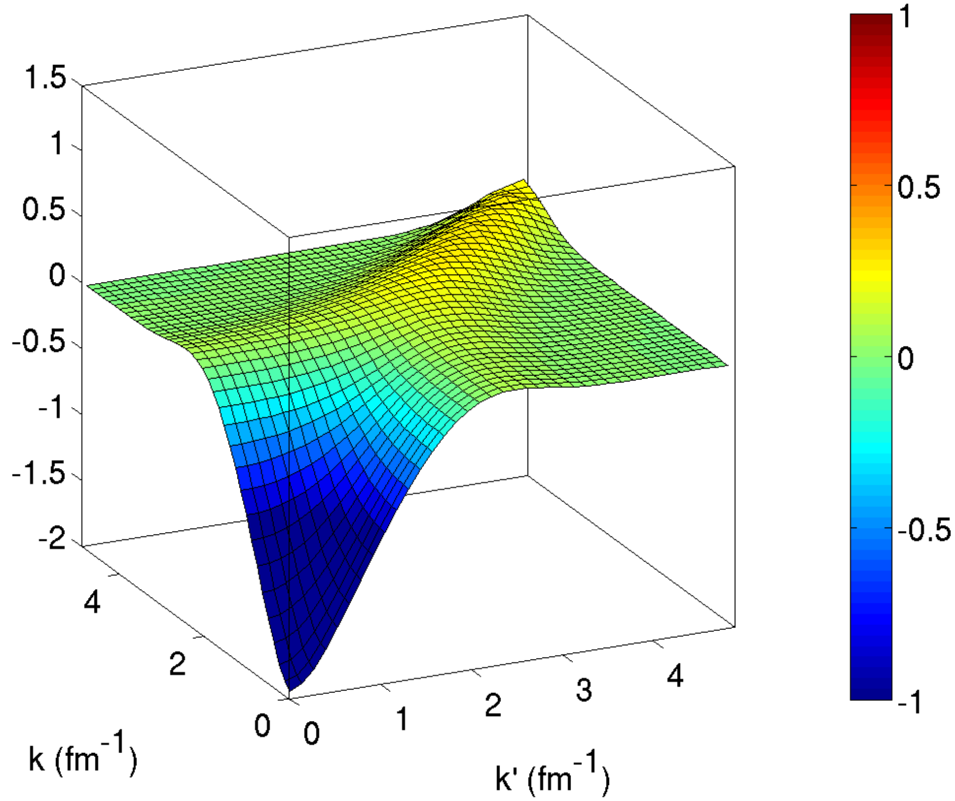
Drive H to band-diagonal form with standard choice:

$$\eta(s) = [T, H(s)]$$

With alternate definition of flow parameter: $\lambda^2 = \frac{1}{\sqrt{s}}$

Argonne V_{18} $^1S_0$





$\lambda = 2.5\,\mathrm{fm}^{-1}$

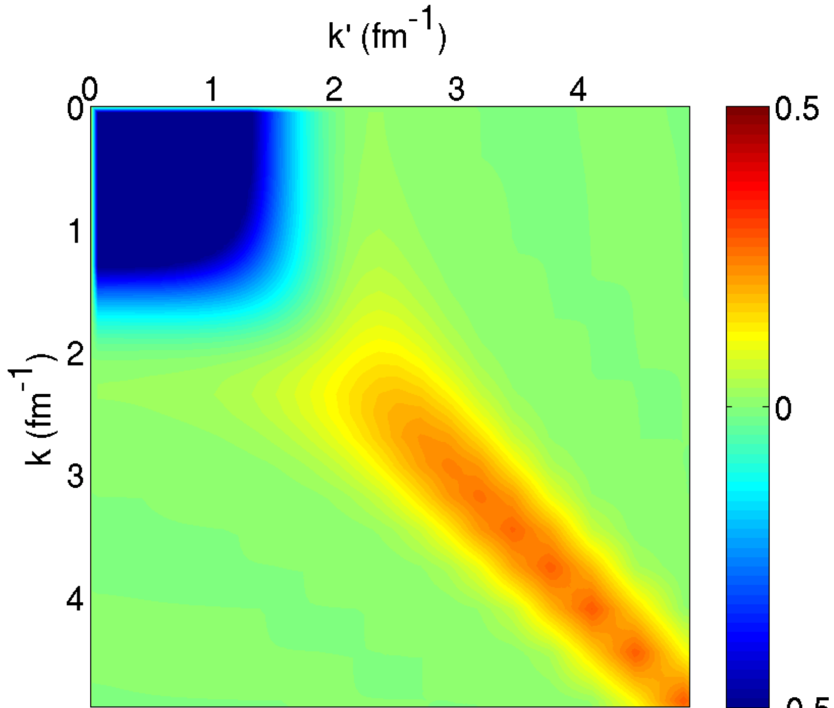
Illustration of SRG Flow

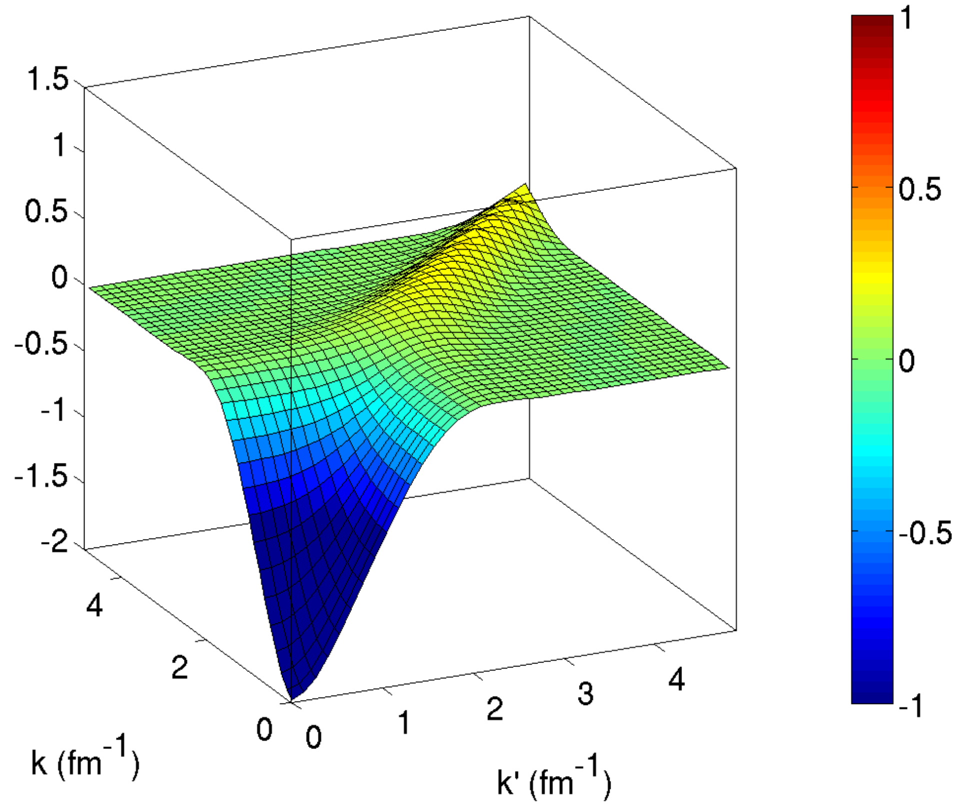
Drive H to band-diagonal form with standard choice:

$$\eta(s) = [T, H(s)]$$

With alternate definition of flow parameter: $\lambda^2 = \dfrac{1}{\sqrt{s}}$

Argonne V_{18} $\ ^1\mathrm{S}_0$





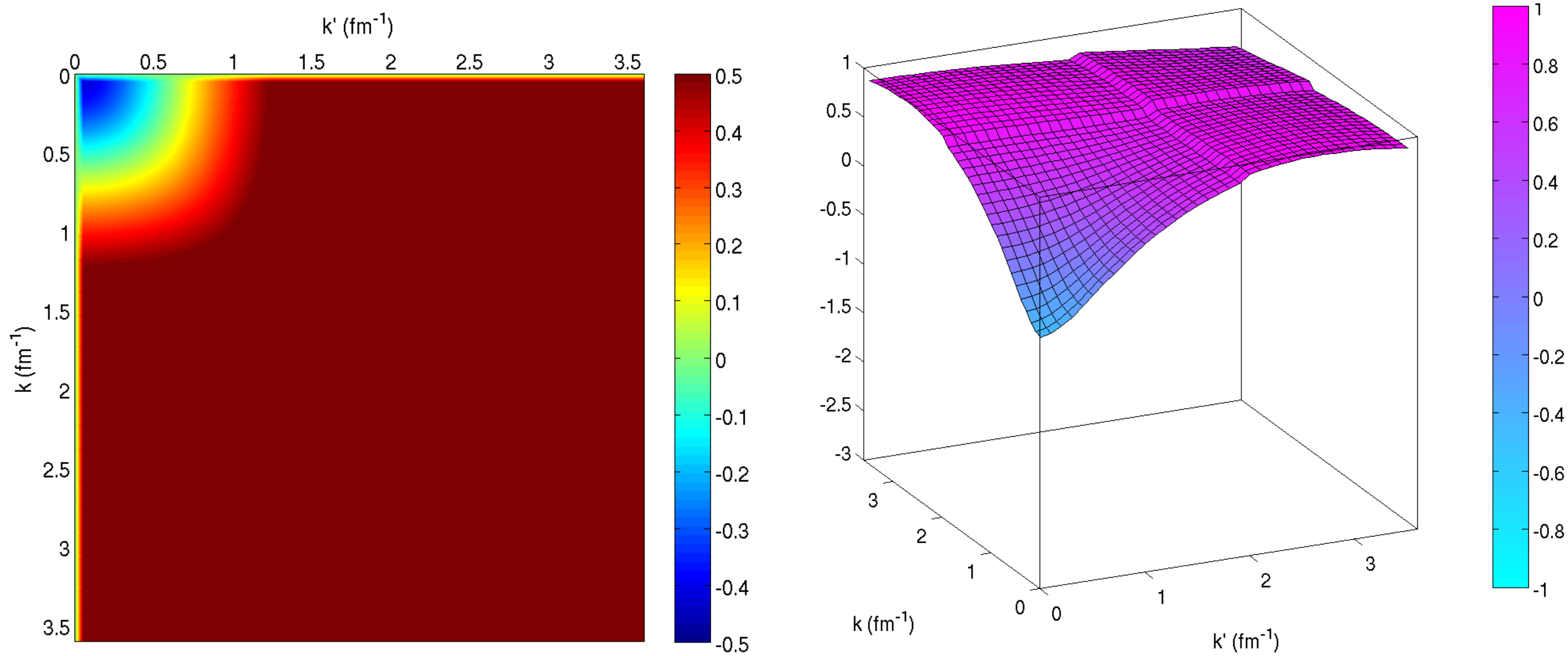
$\lambda = 2.0\,\mathrm{fm}^{-1}$

Other Generator Choices: Block Diagonal

Create block diagonal form like V_{lowk}?

$$G(s) = H_{\text{BD}} = \begin{pmatrix} PH(s)P & 0 \\ 0 & QH(s)Q \end{pmatrix}$$

With alternate definition of flow parameter: $\lambda^2 = \frac{1}{\sqrt{s}}$



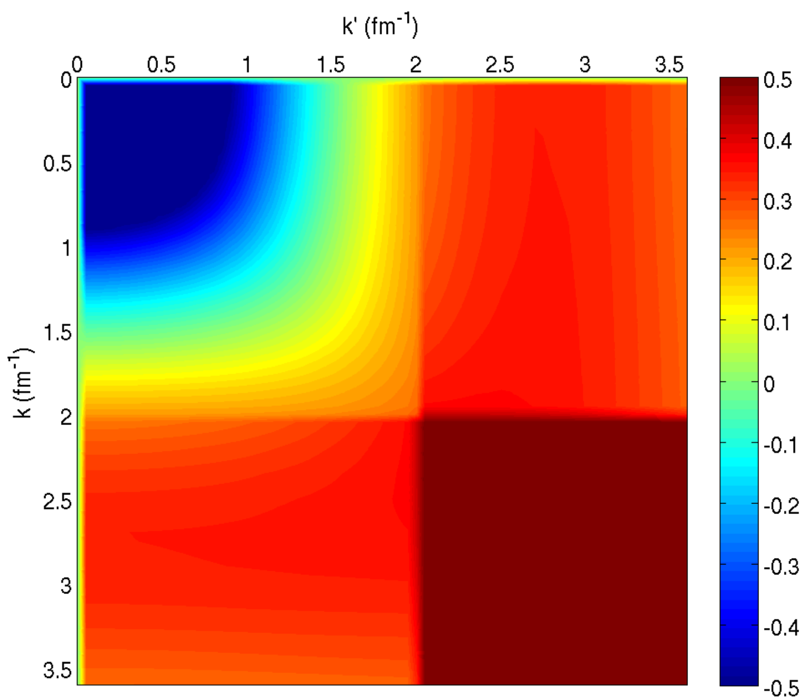
Argonne V_{18} $^3S_1$ $\lambda = 10.0\,\text{fm}^{-1}$

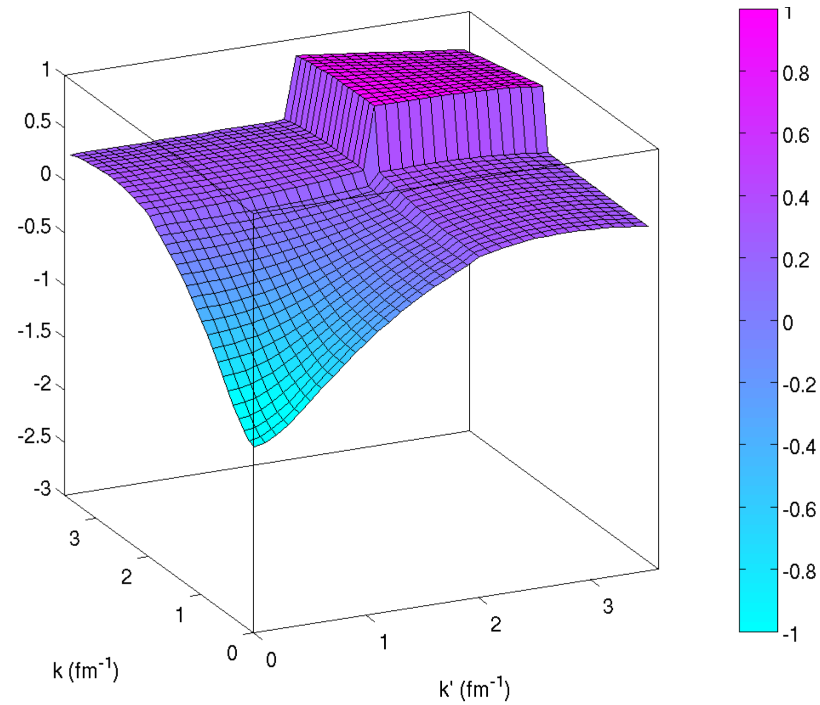
Other Generator Choices: Block Diagonal

Create block diagonal form like V_{lowk}?

$$G(s) = H_{\text{BD}} = \begin{pmatrix} PH(s)P & 0 \\ 0 & QH(s)Q \end{pmatrix}$$

With alternate definition of flow parameter: $\lambda^2 = \dfrac{1}{\sqrt{s}}$





Argonne V_{18} $^3S_1$

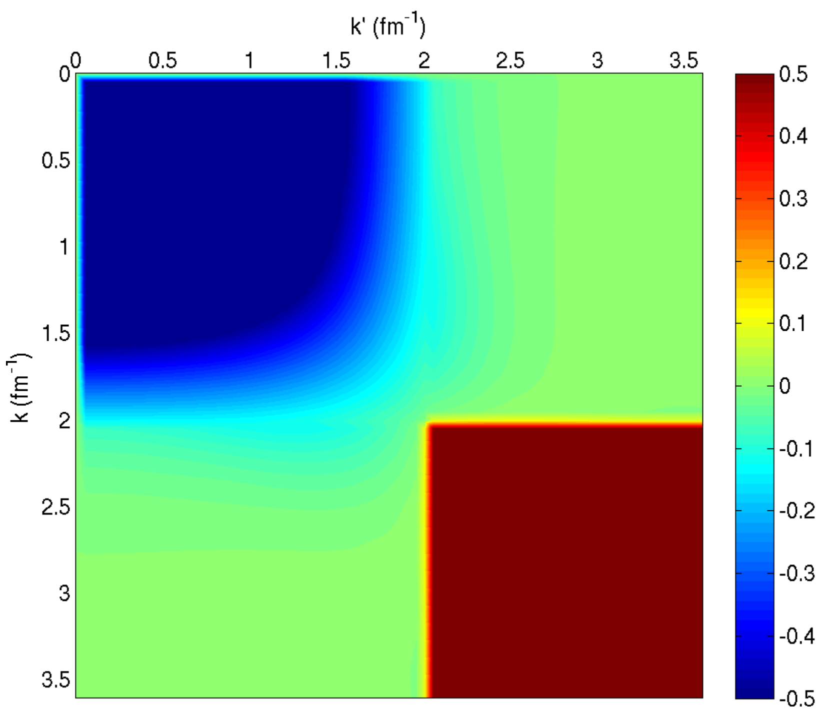
$\lambda = 5.0\,\text{fm}^{-1}$

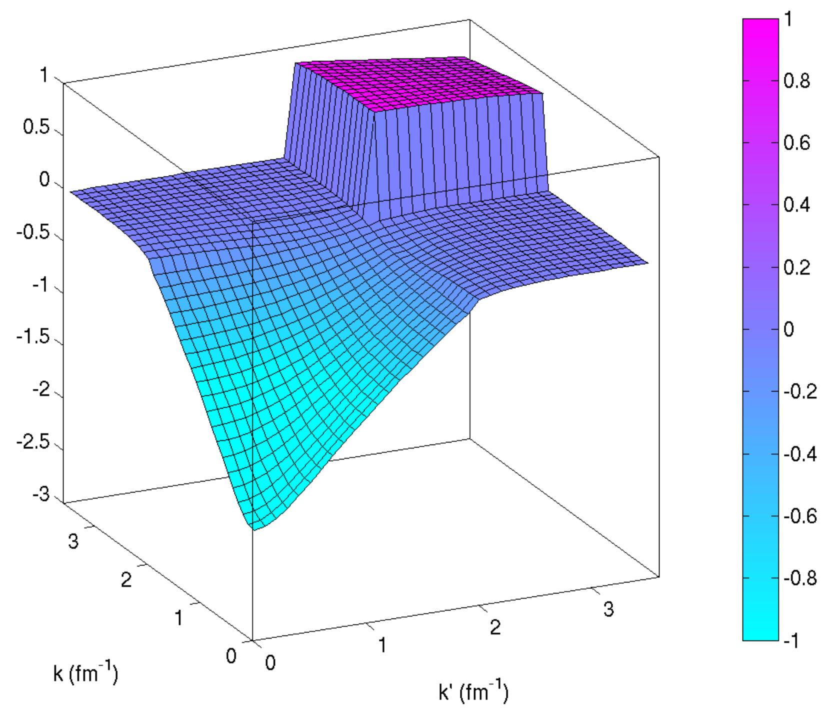
Other Generator Choices: Block Diagonal

Create block diagonal form like V_{lowk}?

$$G(s) = H_{\mathrm{BD}} = \begin{pmatrix} PH(s)P & 0 \\ 0 & QH(s)Q \end{pmatrix}$$

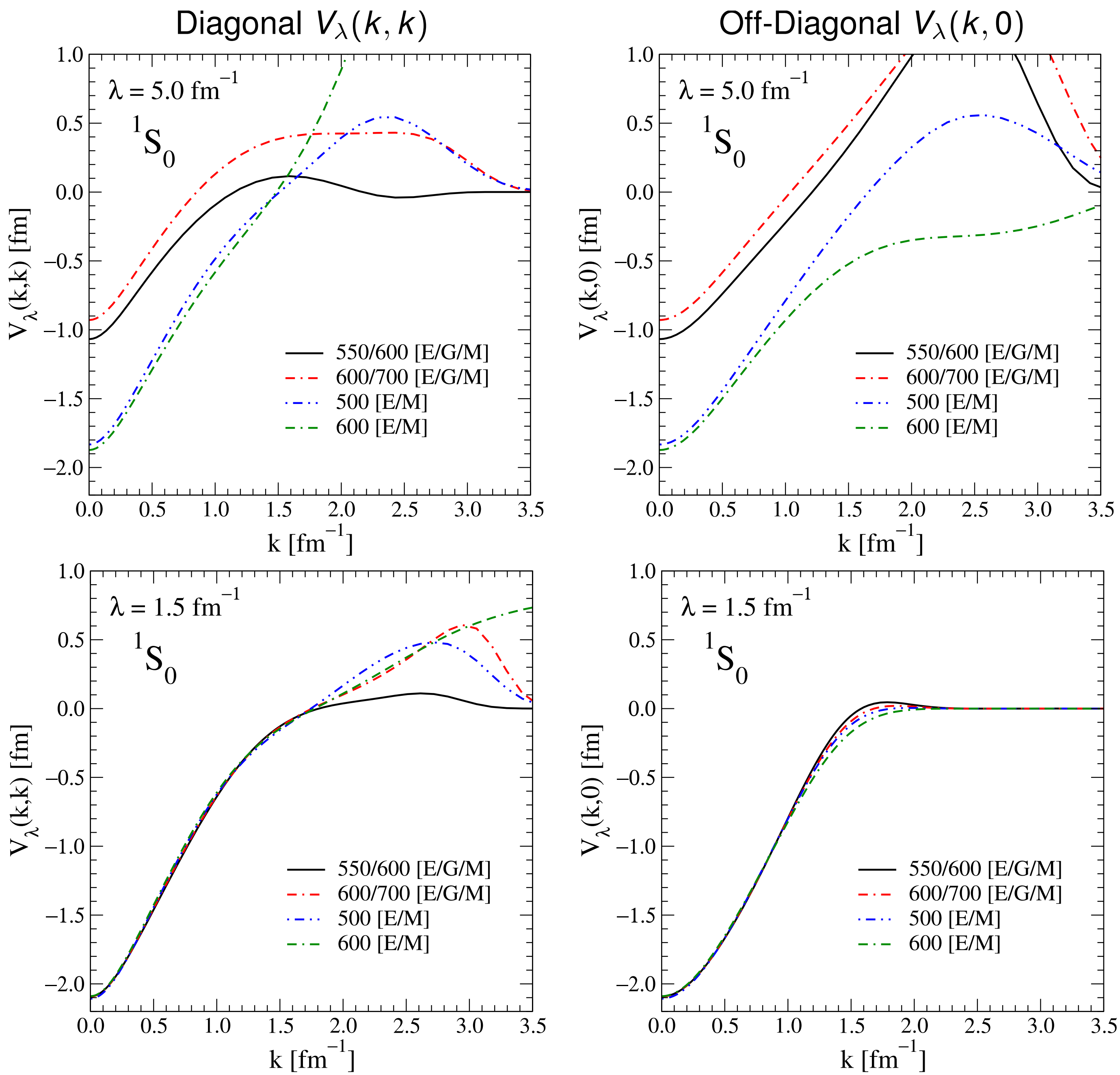
With alternate definition of flow parameter: $\lambda^2 = \dfrac{1}{\sqrt{s}}$





Argonne V_{18} $^3S_1$ $\qquad$ $\lambda = 2.0\,\mathrm{fm}^{-1}$

SRG Renormalization of Chiral EFT Potentials



These are all our favorite Chiral EFT NN potentials…

These are all our favorite Chiral EFT NN potentials… **SRG evolved**

Exhibit similar “universal” behavior as low-momentum interactions!

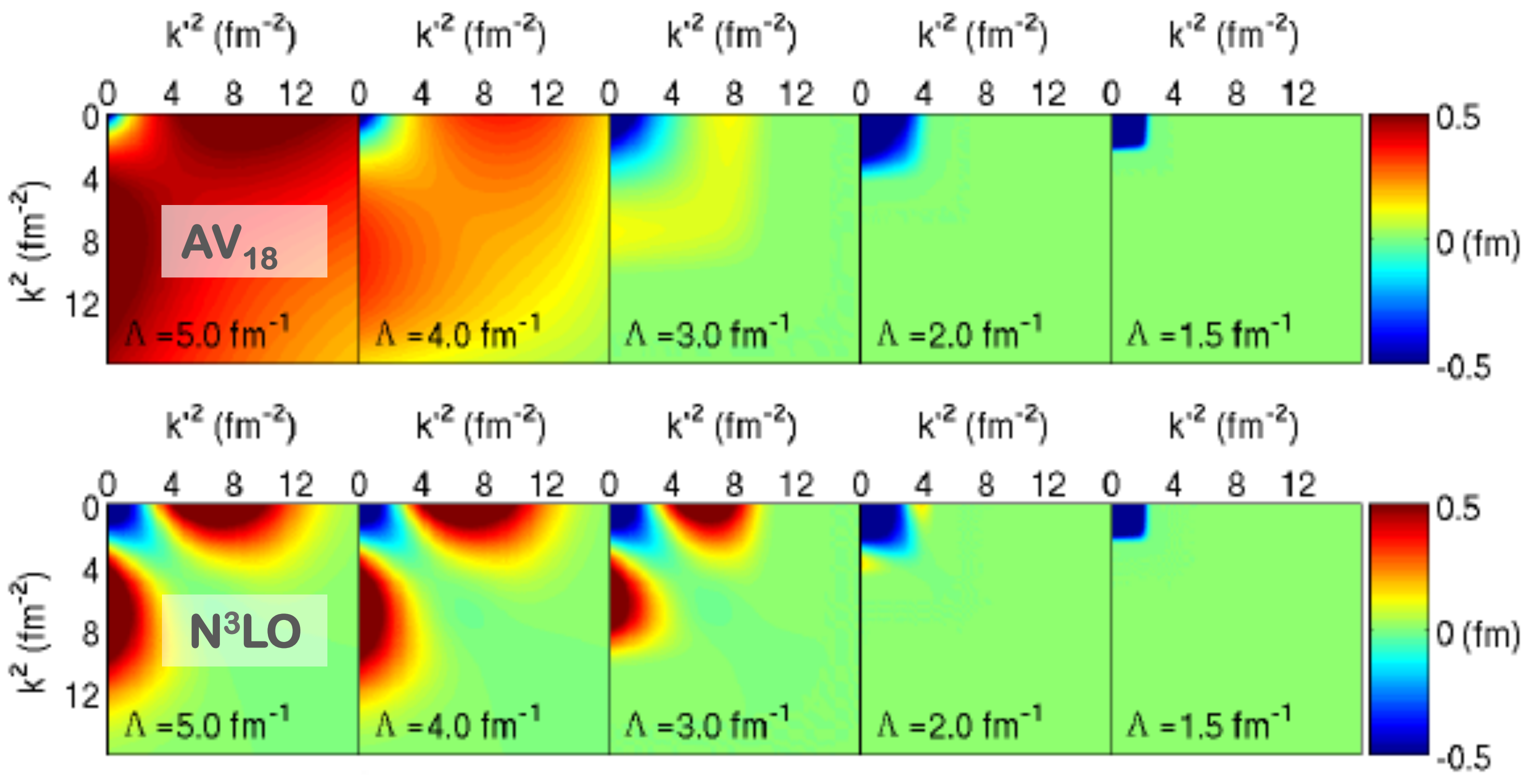
Renormalization of Nuclear Interactions

$$H(\Lambda) = T + V_{\mathrm{NN}}(\Lambda) + V_{3\mathrm{N}}(\Lambda) + V_{4\mathrm{N}}(\Lambda) + \cdots$$

Evolve momentum resolution scale of chiral interactions from initial Λ_χ

Remove coupling to high momenta, low-energy physics unchanged

Bogner, Kuo, Schwenk, Furnstahl



Universal at low-momentum

$V_{\mathrm{low}\,k}(\Lambda)$: lower cutoffs advantageous for nuclear structure calculations

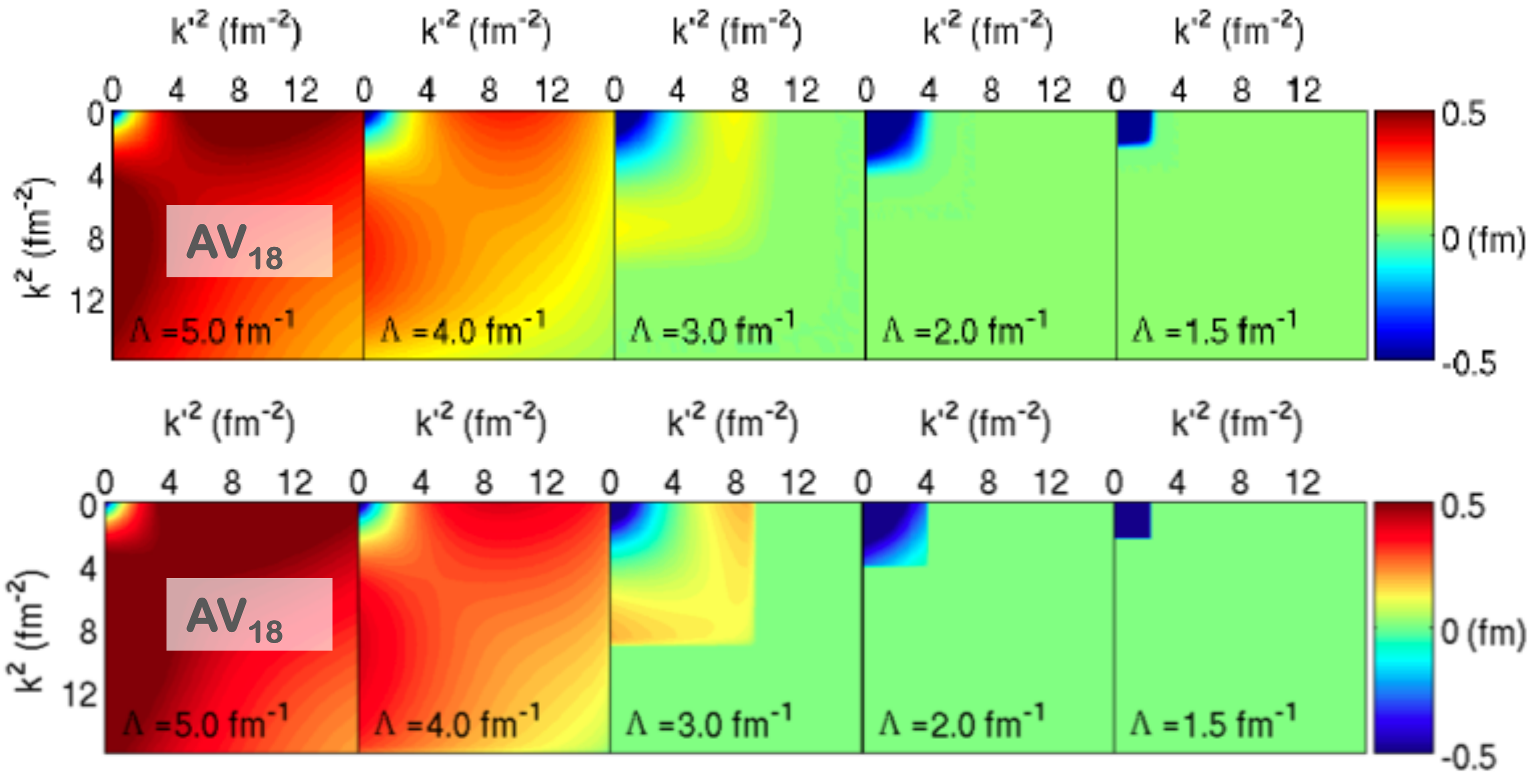
Smooth vs. Sharp Cutoffs

$$H(\Lambda) = T + V_{\mathrm{NN}}(\Lambda) + V_{3\mathrm{N}}(\Lambda) + V_{4\mathrm{N}}(\Lambda) + \cdots$$

Can have sharp as well as smooth cutoffs

Remove coupling to high momenta, low-energy physics unchanged

Bogner, Kuo, Schwenk, Furnstahl

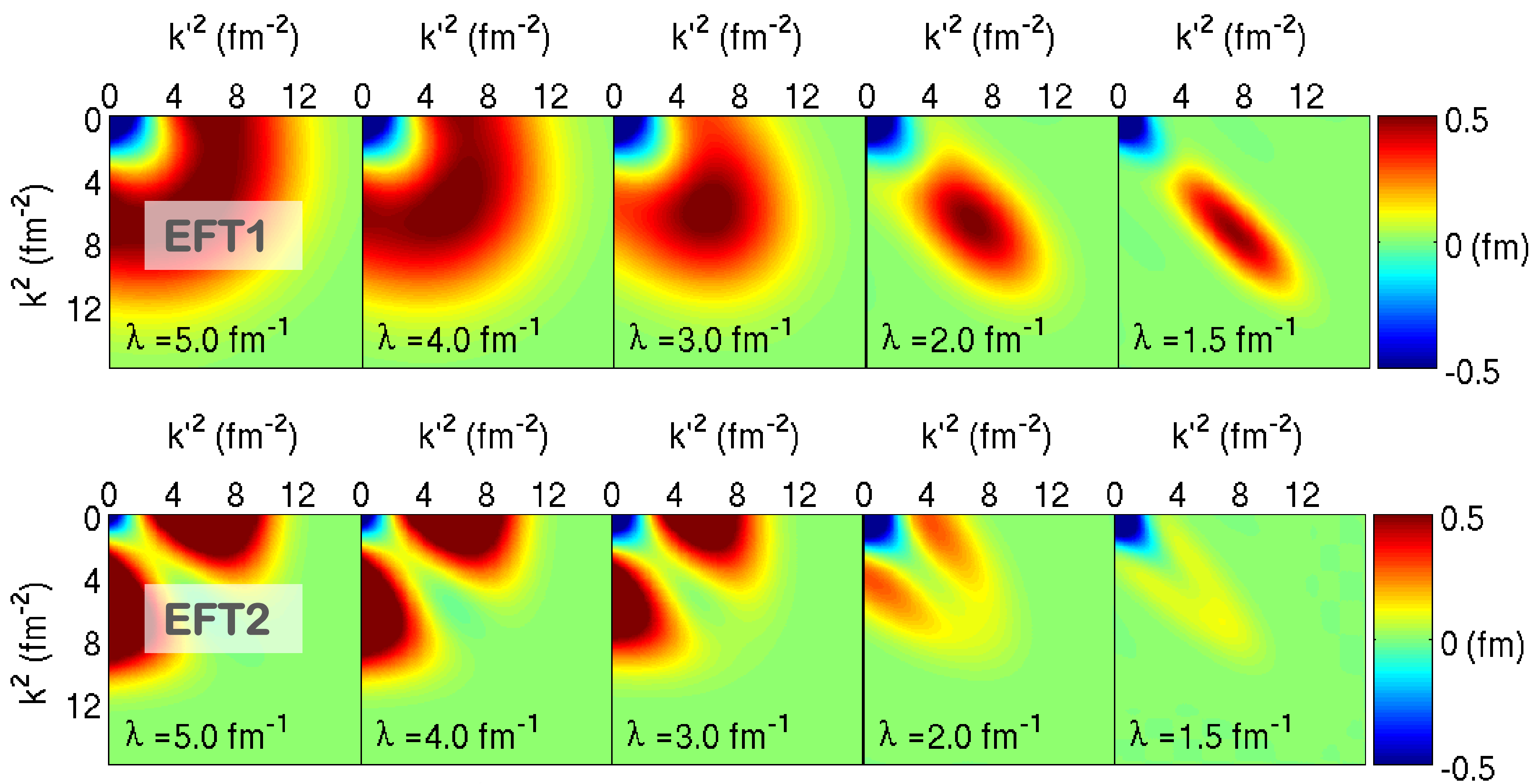


Similar but not exact same results – will be differences in calculations

SRG-Evolution of Different Initial Potentials

$$H(\Lambda) = T + V_{\mathrm{NN}}(\Lambda) + V_{3\mathrm{N}}(\Lambda) + V_{4\mathrm{N}}(\Lambda) + \cdots$$

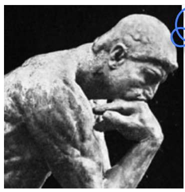
SRG evolution of two different chiral EFT potentials



Lots of pretty pictures, but how does it actually help?

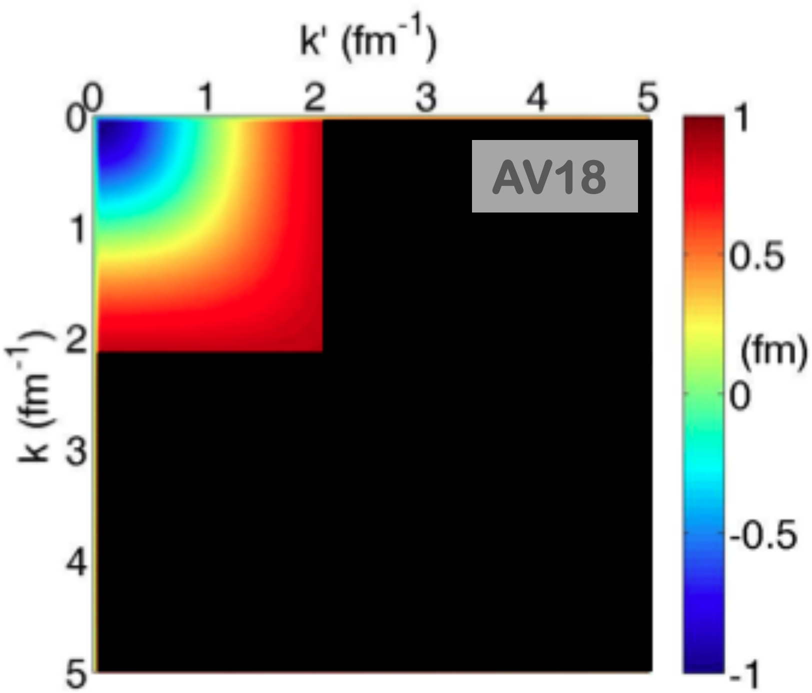
Revisit Low-Pass Filter Idea

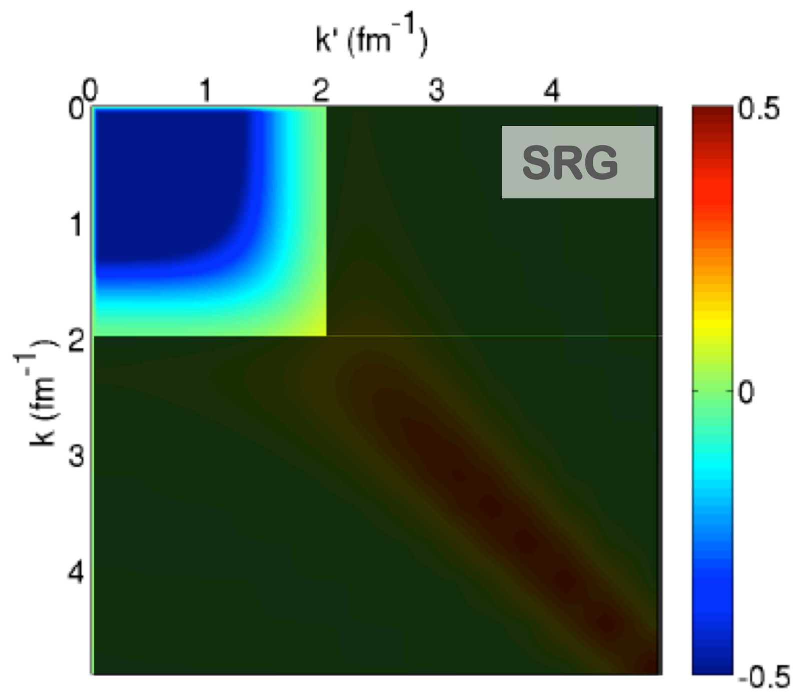
Ok, high momentum is a pain. I wonder what would happen to low-energy observables…



Low-to-high momentum makes life difficult for low-energy nuclear theorists

What's the difference now?

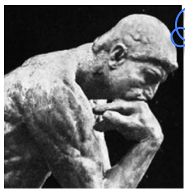




$$V_{\text{filter}}(k', k) \equiv 0; \;\; k, k' > 2.2\,\text{MeV}$$

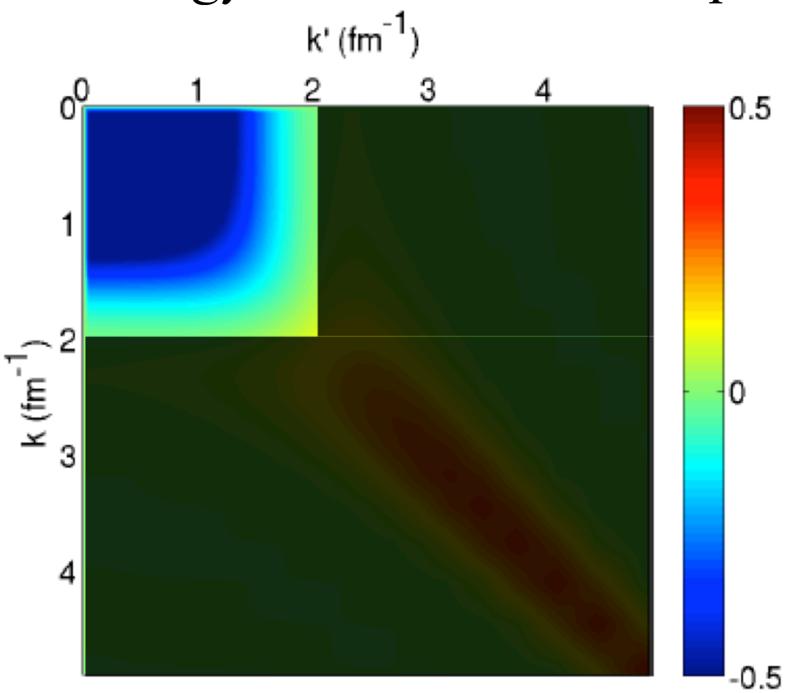
Revisit Low-Pass Filter Idea

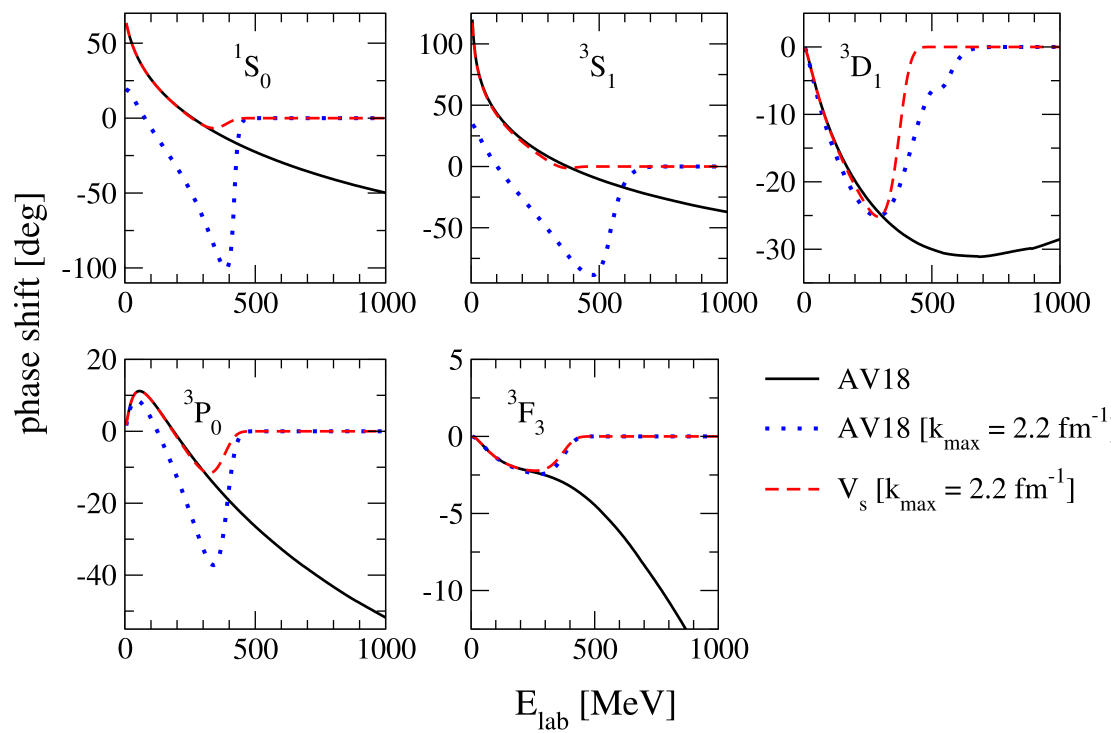
Ok, high momentum is a pain. I wonder what would happen to low-energy observables…



Low-to-high momentum makes life difficult for low-energy nuclear theorists

Low-energy observables were preserved – now sharp cut makes sense!





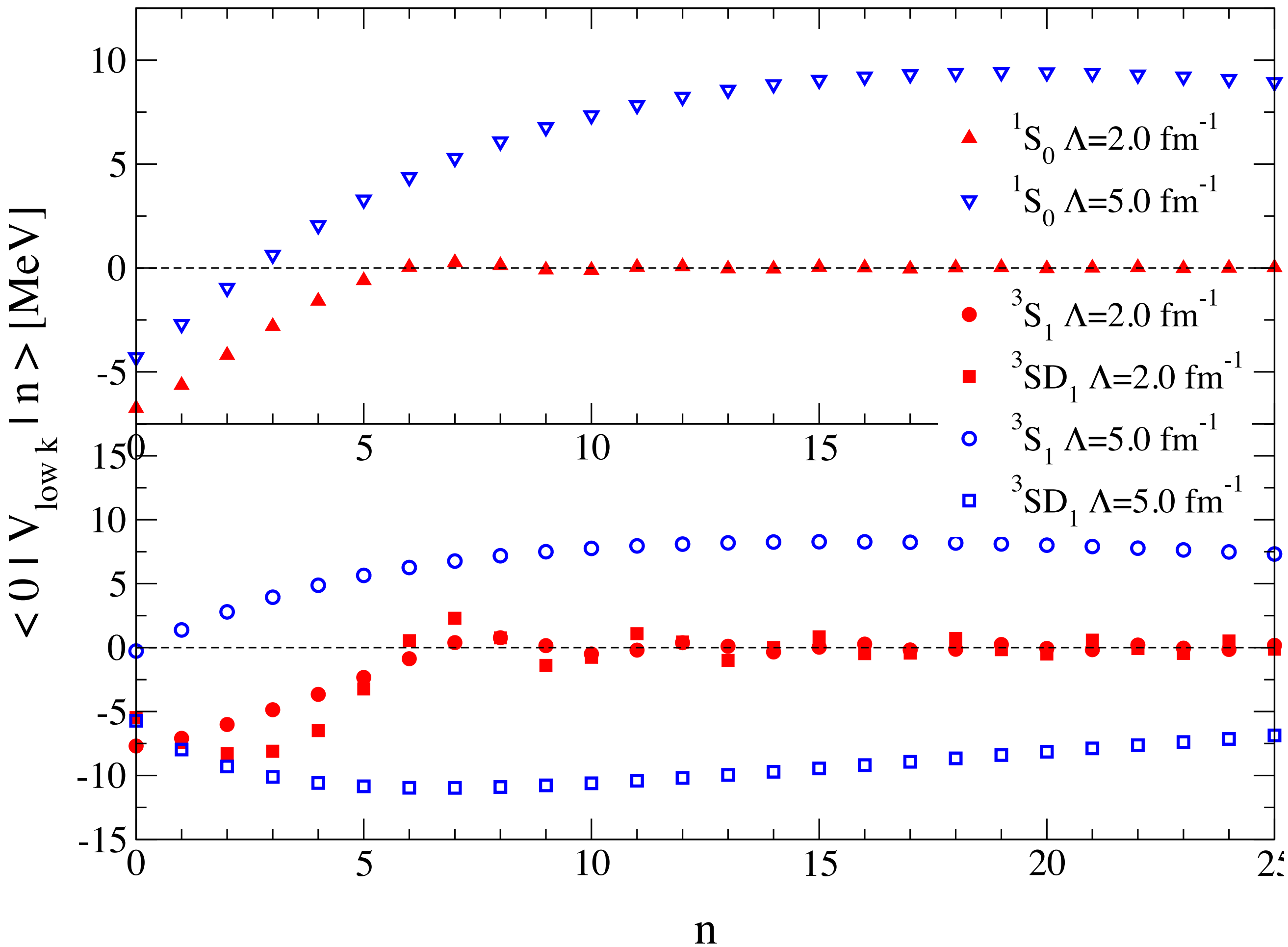
$$V_{\text{filter}}(k', k) \equiv 0;\ \ k, k' > 2.2\,\text{MeV}$$

Benefits of Lower Cutoffs

Often work in HO basis – does this make a difference there?

Removes coupling from low-to-high harmonic oscillator states

Expect to speed convergence in HO basis



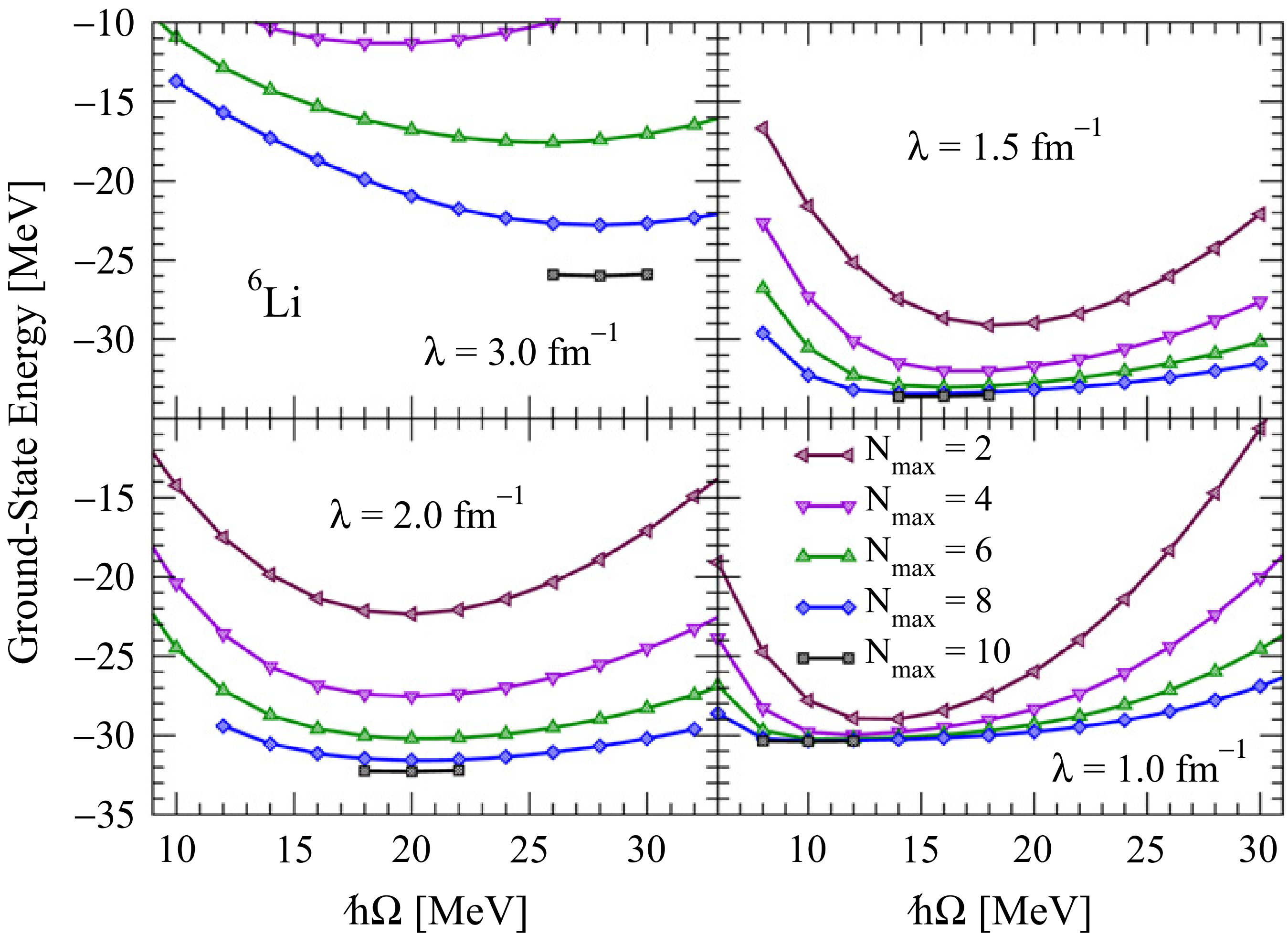
Explicitly see why this causes problems later!

Benefits of Lower Cutoffs

Exactly what happens in **no-core shell model calculations**

Probably equally helpful in normal shell-model calculations?

Come back to this later…



Benefits of Lower Cutoffs

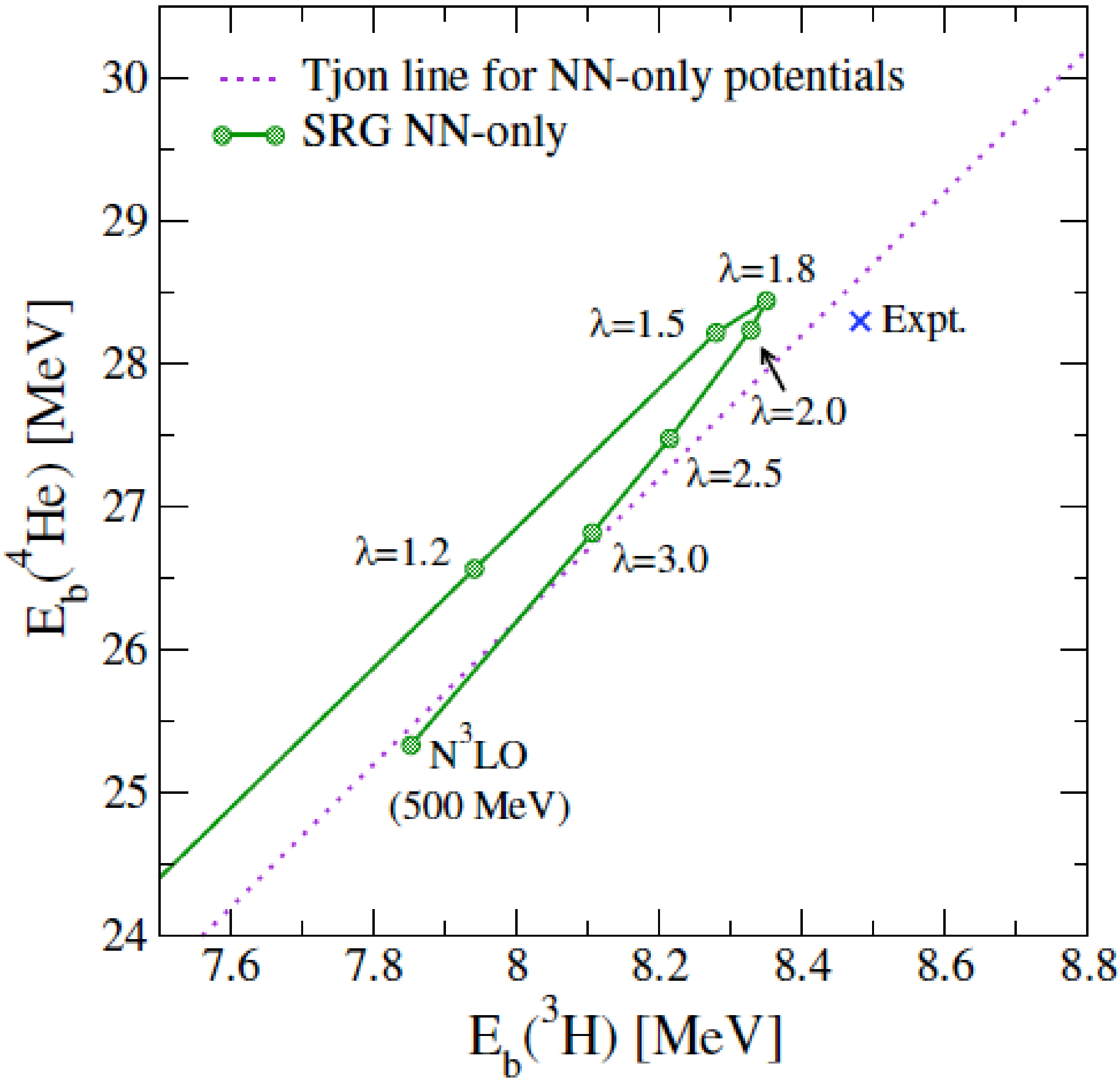
Use cutoff dependence to assess missing physics: return to Tjon line

Varying cutoff moves along line

Still never reaches experiment

Lesson: Variation in physical observables with cutoff indicates missing physics

Tool, not a parameter!

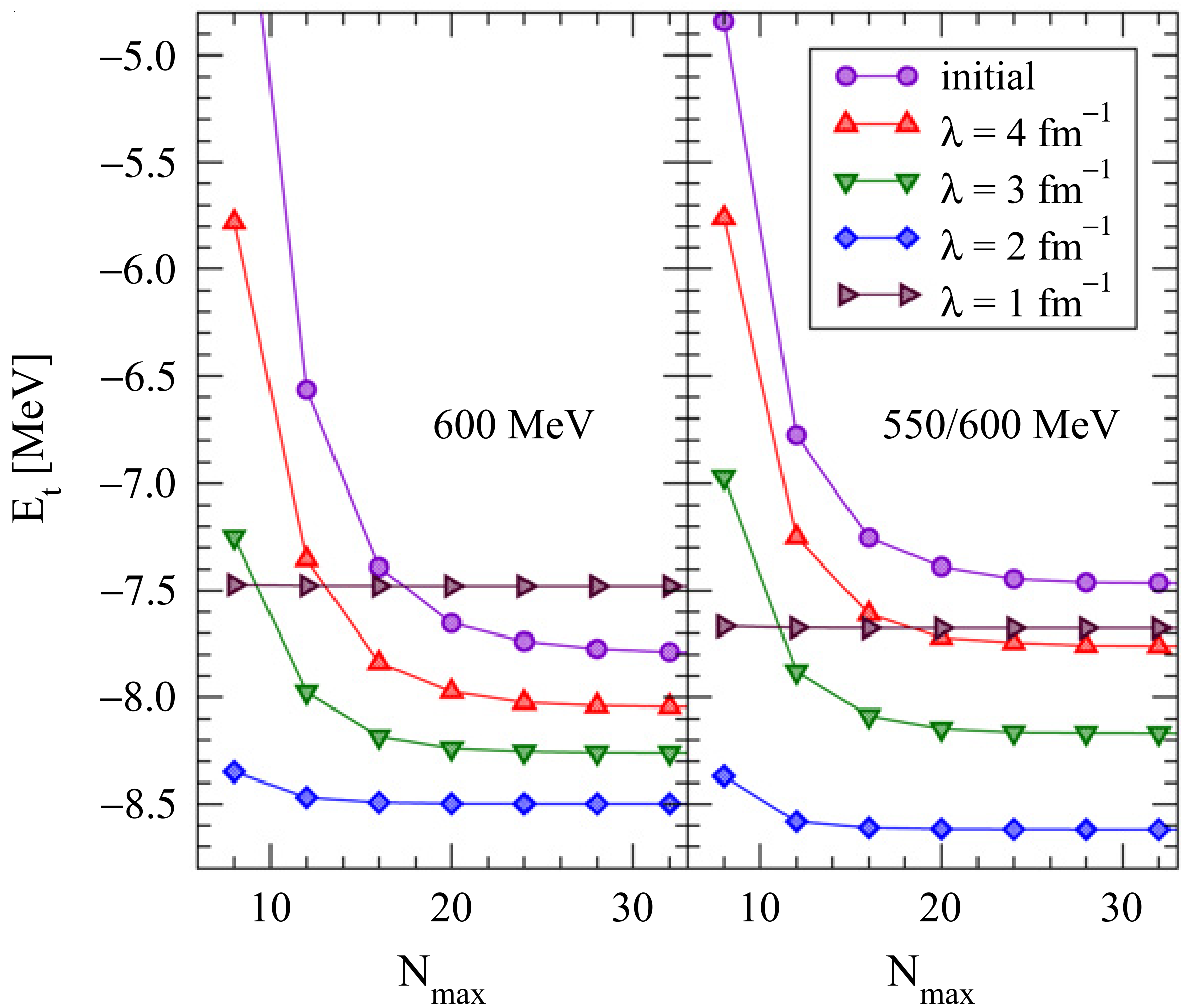


Benefits of Lower Cutoffs

Triton binding energy - again clearly improved convergence behavior

Clear dependence on cutoff – more than one, look closely…

What is the source(s)?



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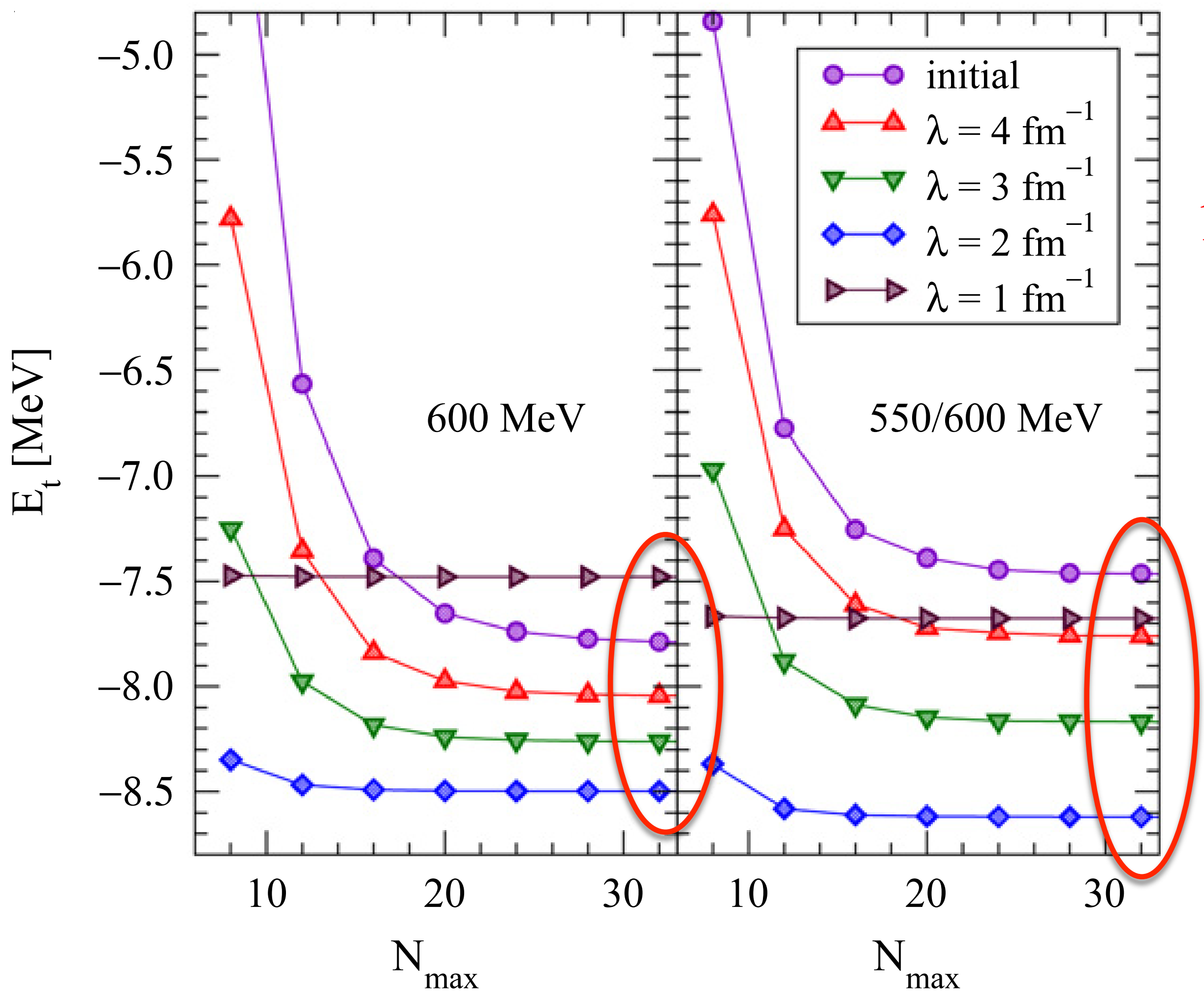

1) SRG cutoff dependence

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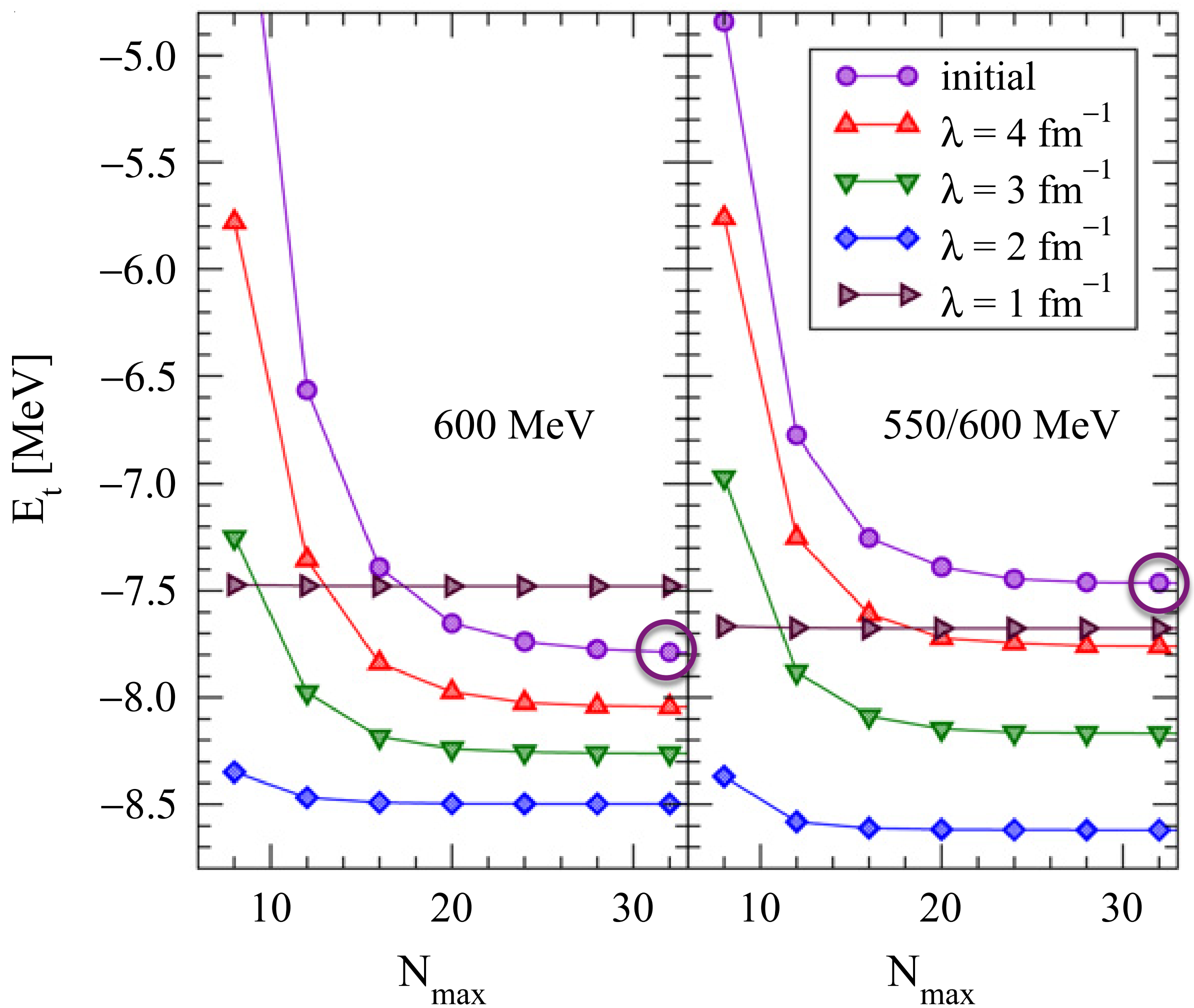

1) SRG cutoff dependence

2) Initial cutoff dependence

Something missing in each case!

Case 1: Price of Low Cutoffs = Induced Forces

Life Lesson: no free lunch – not even at Summer Schools, apparently ☹

Consider Hamiltonian with only two-body forces:

$$H = T + V_{\mathrm{NN}}$$

And $\eta(s) = [T, H(s)]$

$$\frac{\mathrm{d}H(s)}{\mathrm{d}s} = [\eta(s), H(s)] = [[T, T + V(s)], T + V(s)]$$

Simply expand with creation/annihilation operators:

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Simply expand with creation/annihilation operators:

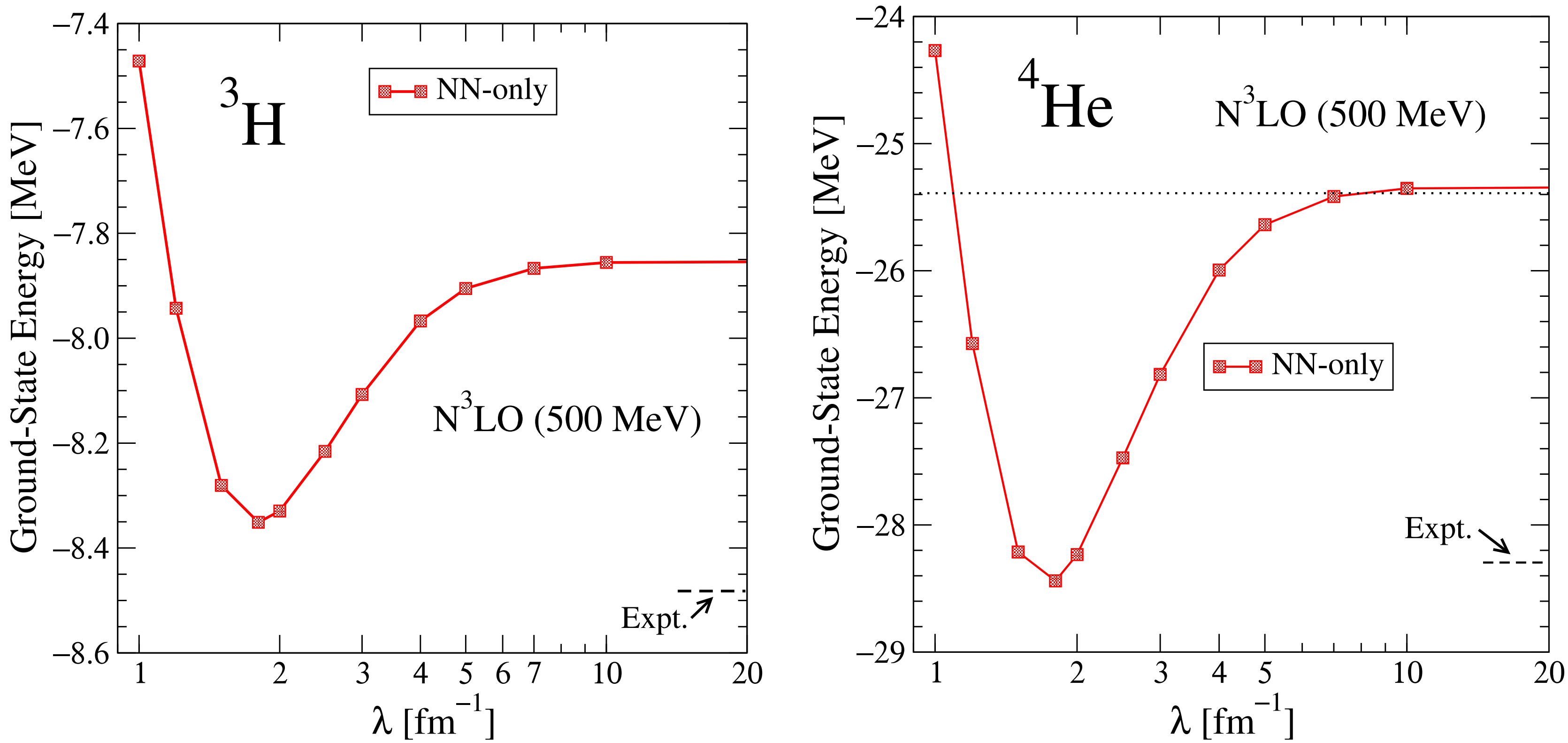
$$\frac{\mathrm{d}V(s)}{\mathrm{d}s} = \left[\left[\sum a^\dagger a, \sum a^\dagger a^\dagger a a\right], \sum a^\dagger a^\dagger a a\right] = \cdots + \sum a^\dagger a^\dagger a^\dagger a a a + \cdots$$

Three-body terms will appear even when initial 3-body forces absent

Call these **induced 3N forces (3N-ind)**

Induced 3N Forces

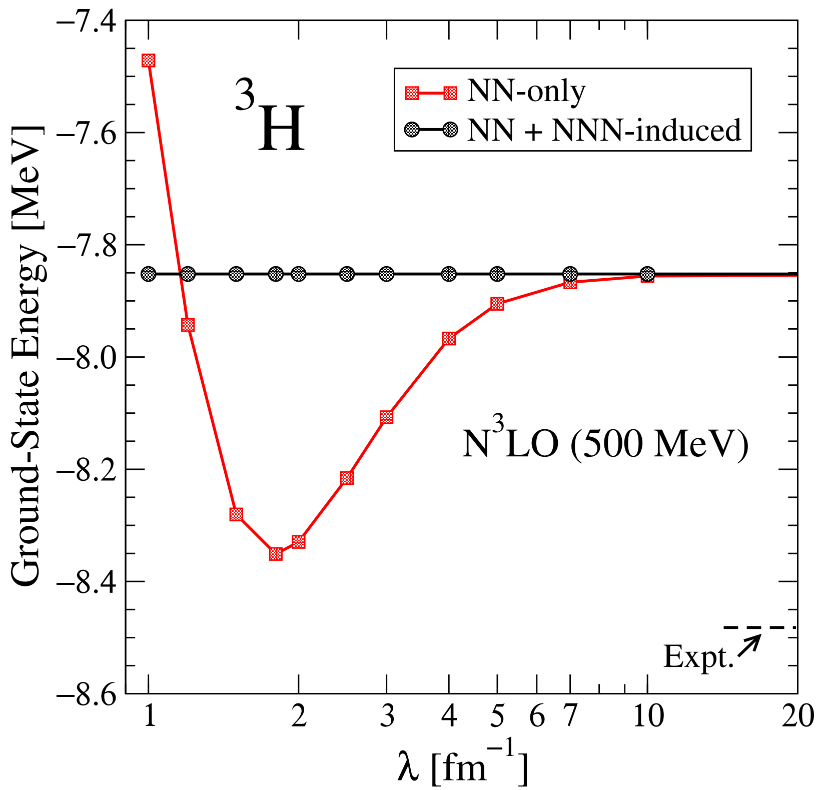
Effect of including 3N-ind? Exactly initial $V_{\rm NN}$ **up to neglected 4N-ind**

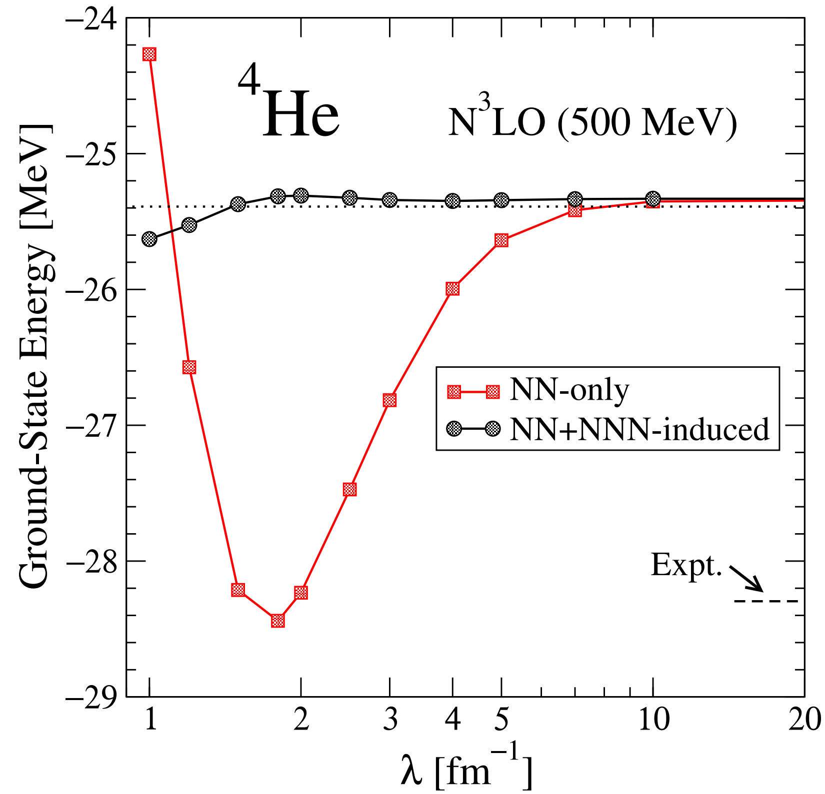


NN-only clear cutoff dependencs

Induced 3N Forces

Effect of including 3N-ind? Exactly initial $V_{\rm NN}$ **up to neglected 4N-ind**





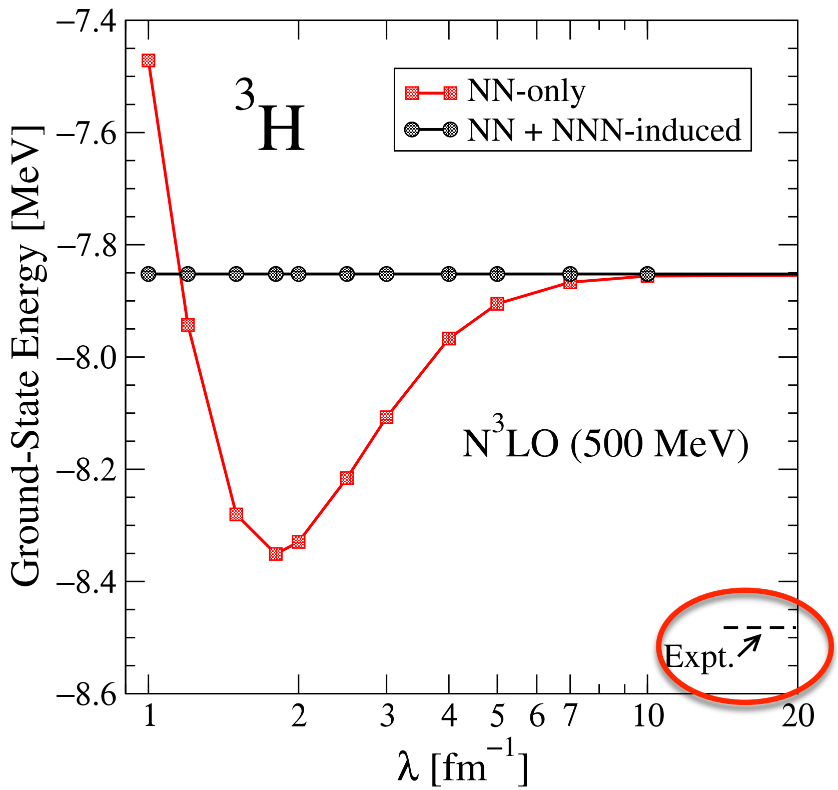
NN-only clear cutoff dependencs

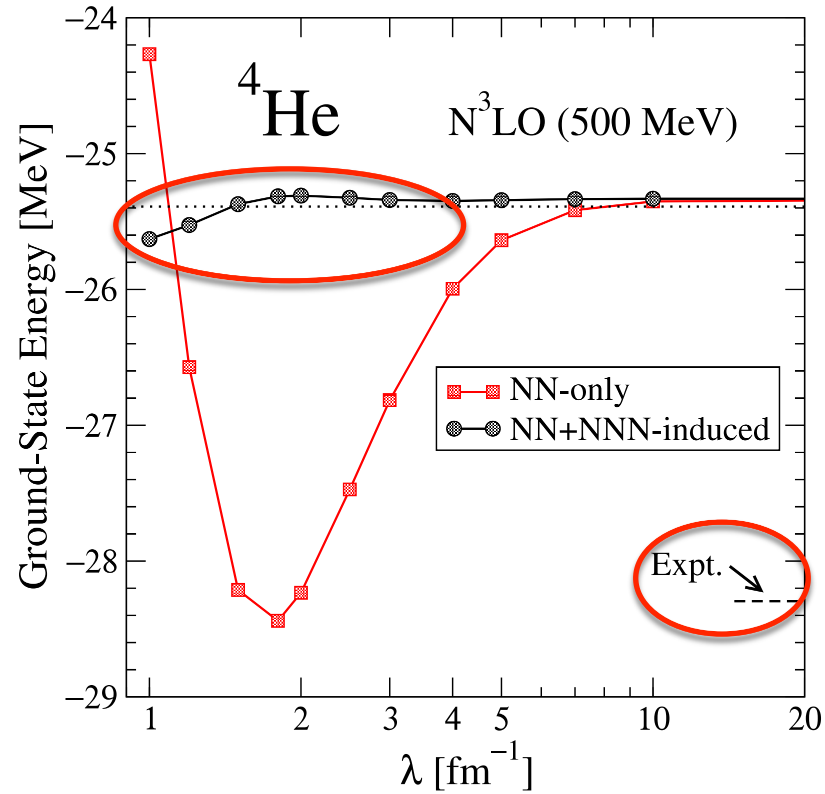
3N-induced – dramatic reduction in cutoff dependence!

Lesson: SRG cutoff variation a sign of neglected induced forces

Induced 3N Forces

Effect of including 3N-ind? Exactly initial V_{NN} **up to neglected 4N-ind**





NN-only clear cutoff dependencs

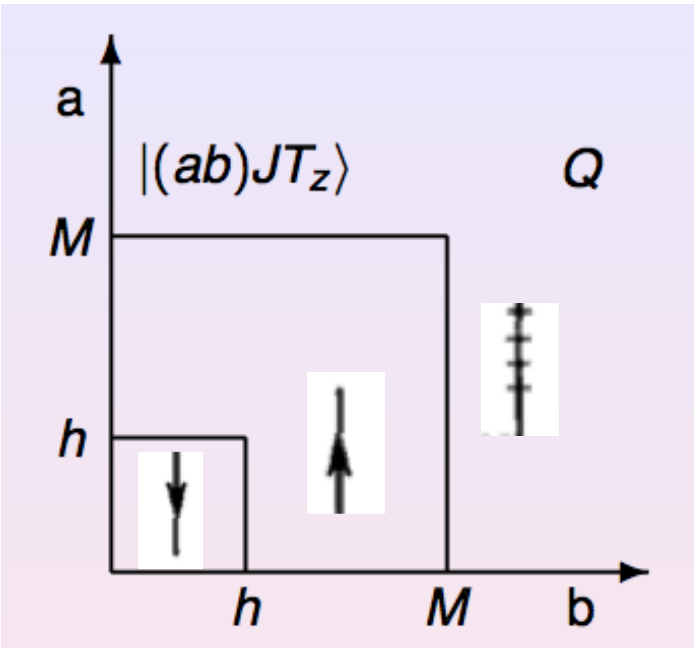
3N-induced – dramatic reduction in cutoff dependence!

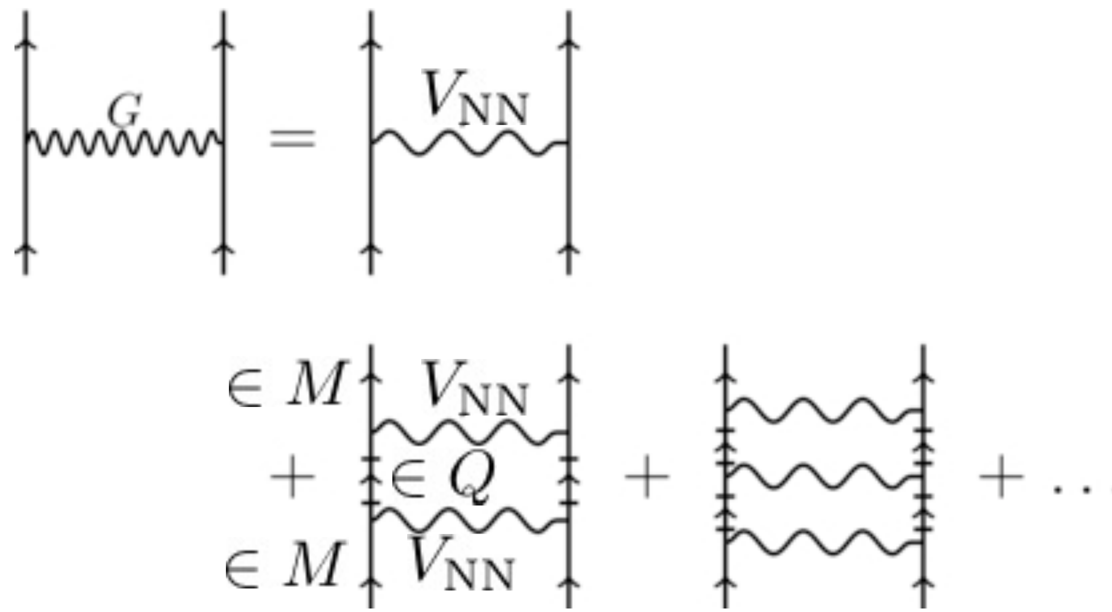
Lesson: SRG cutoff variation a sign of neglected induced forces

Still far from experiment and remaining (minor) cutoff dependence!

Aside: G-matrix Renormalization

Standard method for softening interaction in nuclear structure for decades:





Infinite summation of ladder diagrams

Need two model spaces:

1) **M** space in which we will want to calculate (excitations allowed in M)

2) Large space **Q** in which particle excitations are allowed

To avoid double counting, can't overlap – **matrix elements depend on M**

Aside: G-matrix Renormalization

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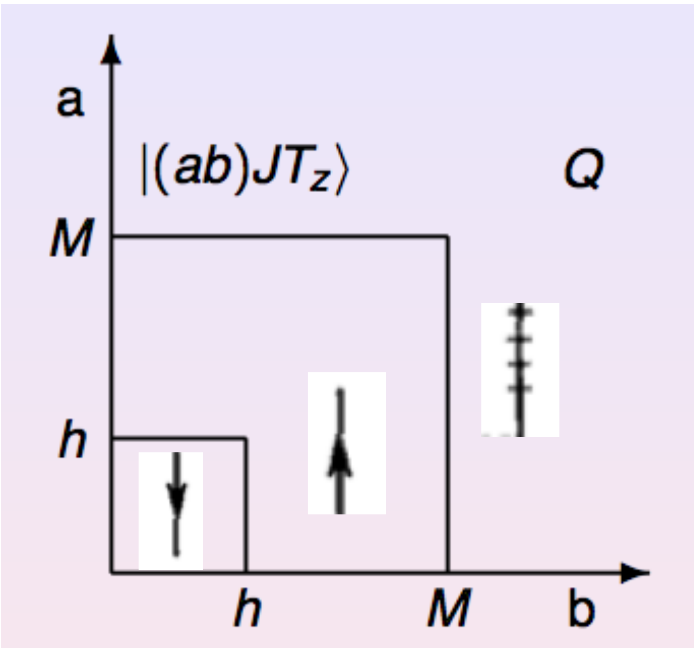

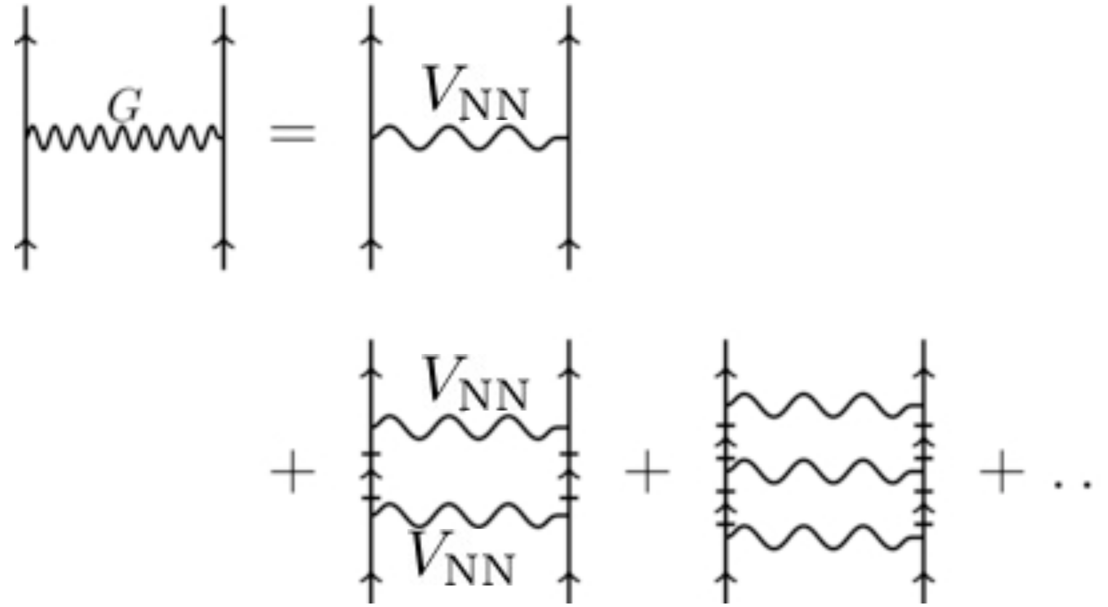

$$G_{ijkl}(\omega) = V_{ijkl} + \sum_{mn \in Q} V_{ijmn} \frac{Q}{\omega - \varepsilon_m - \varepsilon_n} G_{mnkl}(\omega)$$

Iterative procedure

Dependence on arbitrary starting energy!

G-matrix Renormalization

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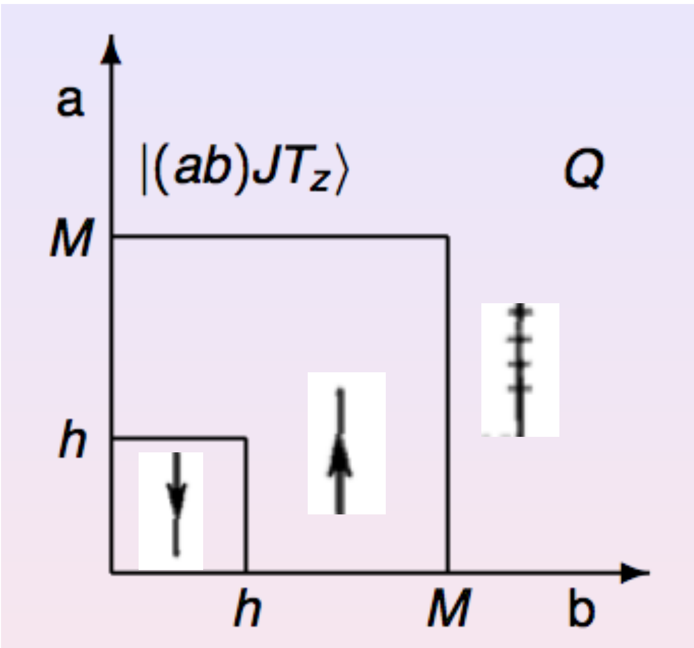

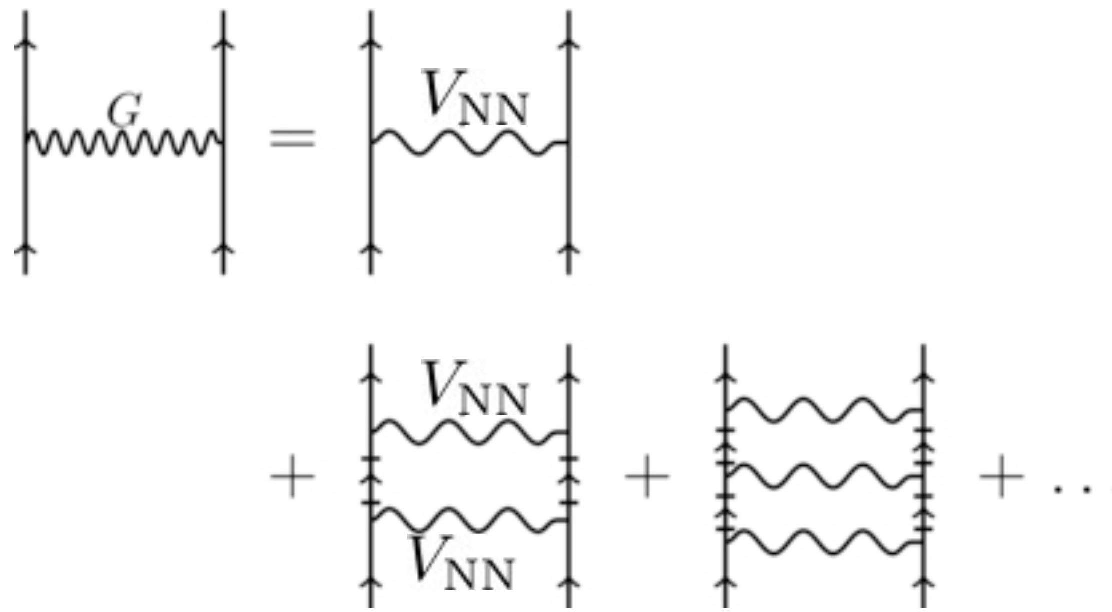

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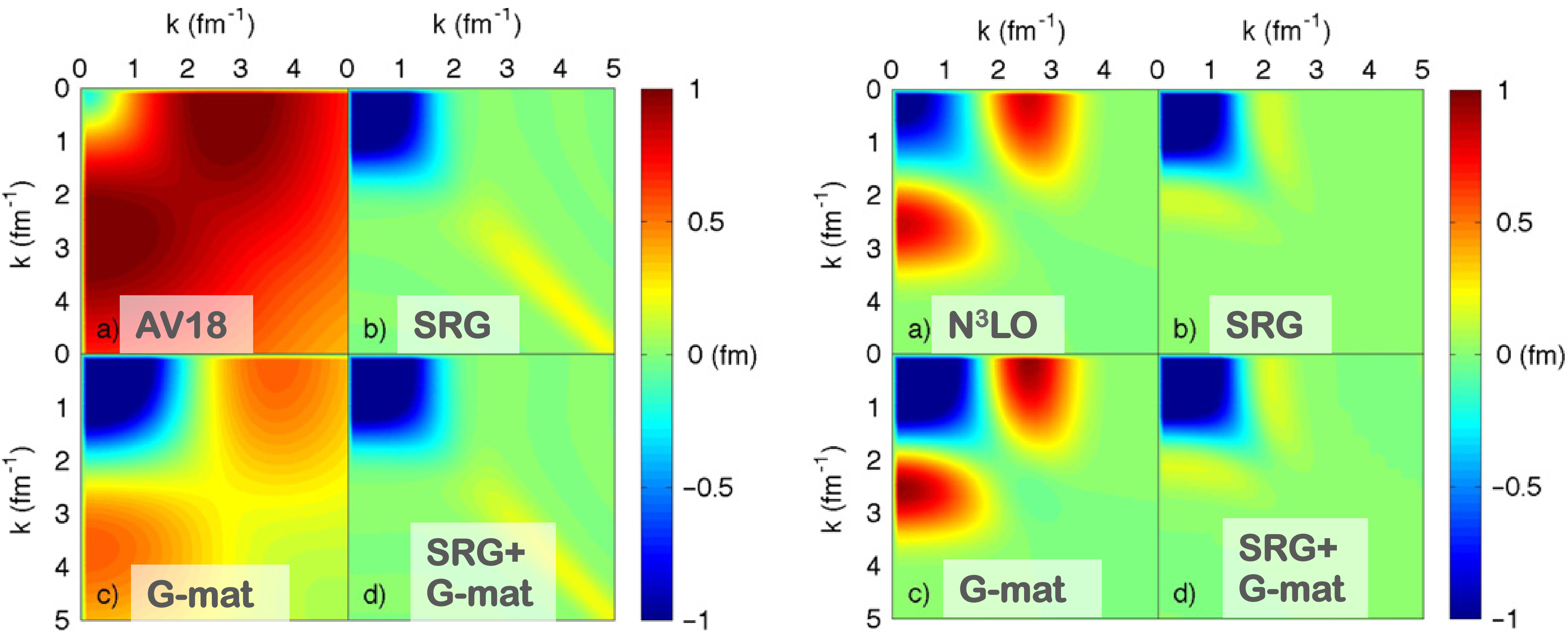
G-matrix Renormalization

Results of **G-matrix** renormalization vs. SRG



Removes some diagonal high-momentum components

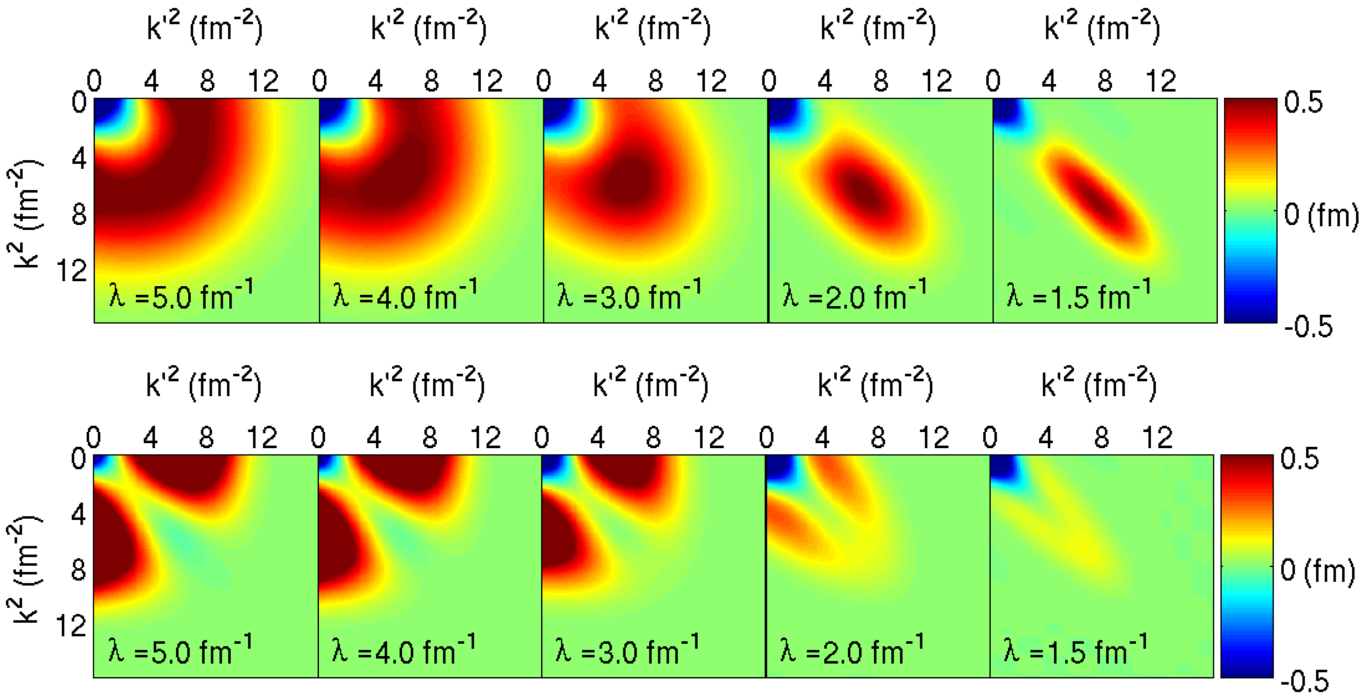
Still large low-to-high coupling in both interactions

No indication of universality

Clear difference compared with SRG-evolved interactions!

Summary

Low-momentum interactions can be constructed from any V_{NN} via RG



Low-to-high momentum coupling not desirable in low-energy nuclear physics

Evolve to low-momentum while preserving low-energy physics

Universality attained near cutoff of data

Low-momentum cutoffs remove low-to-high harmonic oscillator couplings

Cutoff variation assesses missing physics interaction level: tool not a parameter