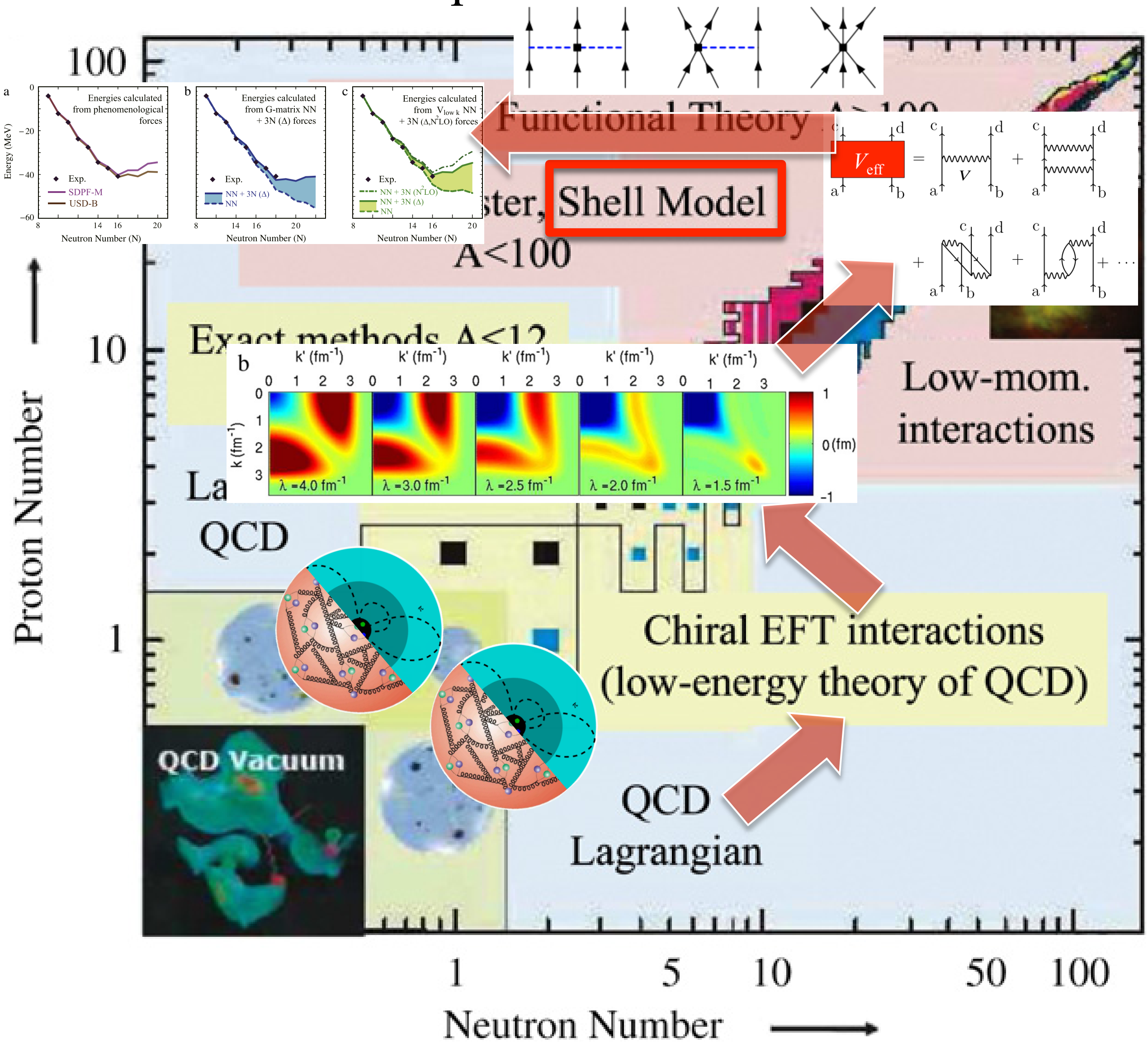
Part IV: Three-Nucleon Forces to Nuclei

To understand the properties of complex nuclei from first principles



Three-Nucleon Forces

Basic ideas – why needed?

3N from chiral EFT

Implementing in shell model

Relation to monopoles

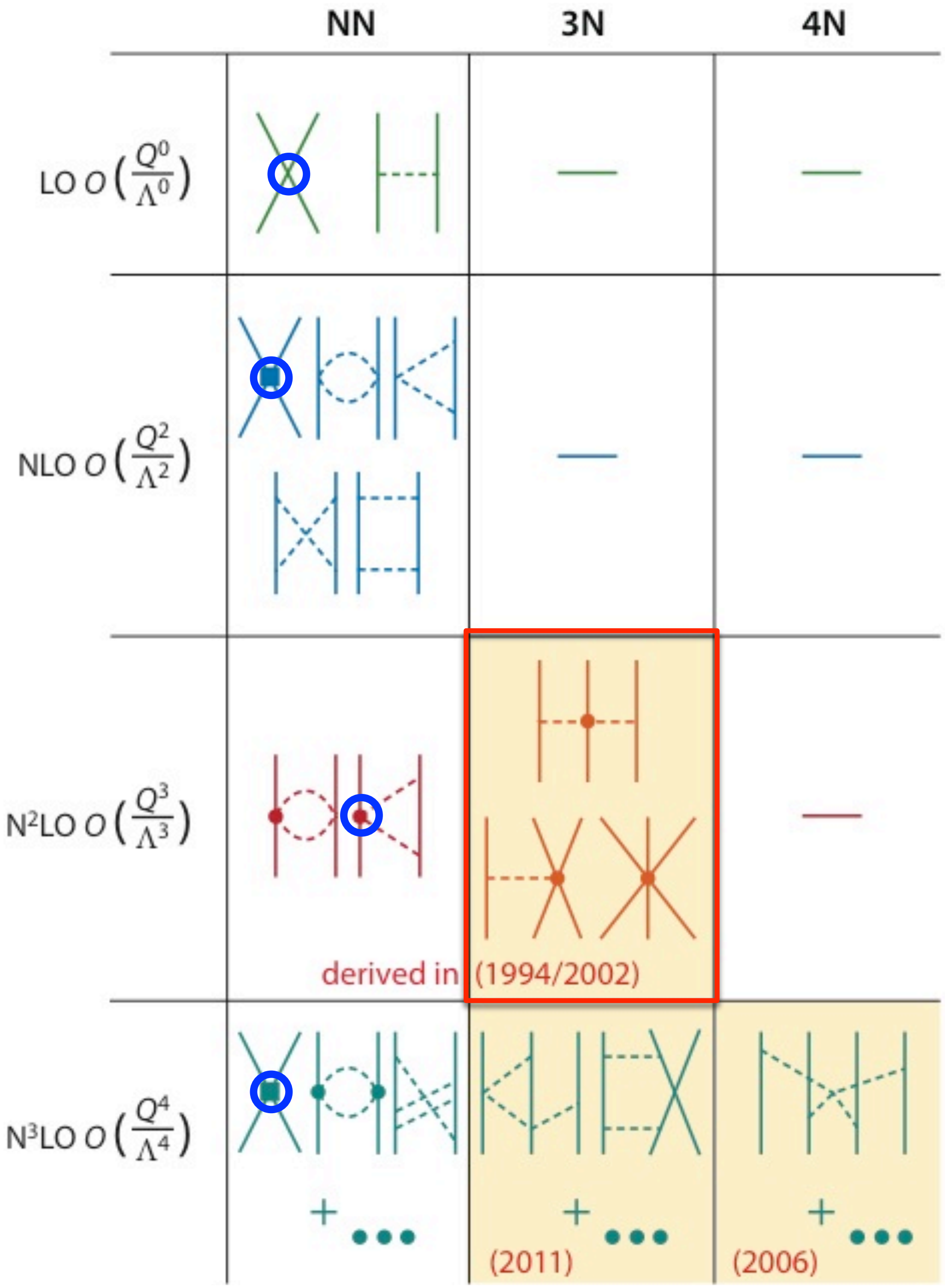
Predictions/new discoveries

Connections beyond structure

How will we approach this problem:

QCD → NN (3N) forces → Renormalize → "Solve" many-body problem → Predictions

Chiral Effective Field Theory: Nuclear Forces

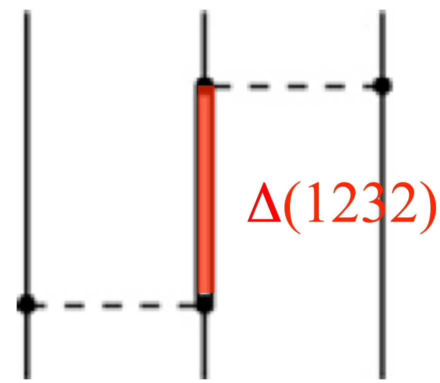


Weinberg, van Kolck, Kaplan, Savage, Wise

Nucleons interact via pion exchanges and contact interactions

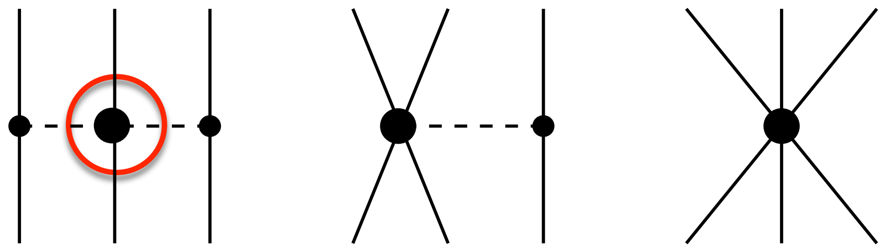
Consistent treatment of NN, 3N,…

NN couplings fit to scattering data



Chiral EFT: N²LO 3N

First non-vanishing 3N contributions: Next-to-next-to-leading order $\nu = 3$



$$V_{3N}^{(3)} = \frac{g_A^2}{8F_\pi^4} \frac{\vec{\sigma}_1 \cdot \vec{q}_1 \vec{\sigma}_3 \cdot \vec{q}_3}{[q_1^2 + M_\pi^2][q_3^2 + M_\pi^2]} [\boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_3 (-4 c_1 M_\pi^2 + 2 c_3 \vec{q}_1 \cdot \vec{q}_3) + c_4 \boldsymbol{\tau}_1 \times \boldsymbol{\tau}_3 \cdot \boldsymbol{\tau}_2 \vec{q}_1 \times \vec{q}_3 \cdot \vec{\sigma}_2] - \frac{g_A D}{8F_\pi^2} \frac{\vec{\sigma}_3 \cdot \vec{q}_3}{q_3^2 + M_\pi^2} \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_3 \vec{\sigma}_1 \cdot \vec{q}_3 + \frac{1}{2} E \boldsymbol{\tau}_2 \cdot \boldsymbol{\tau}_3$$

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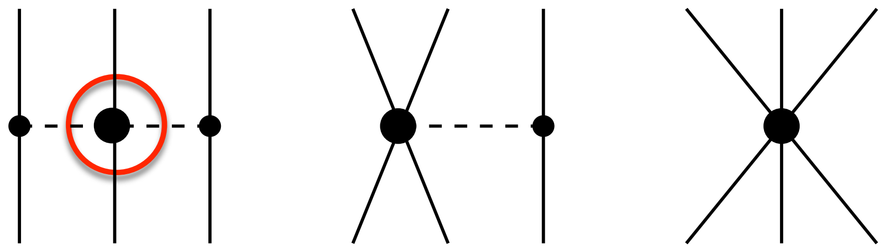
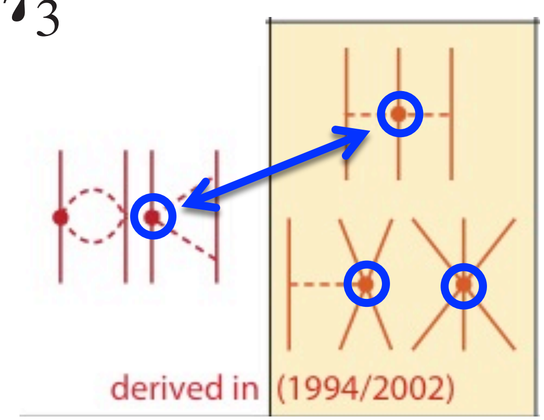


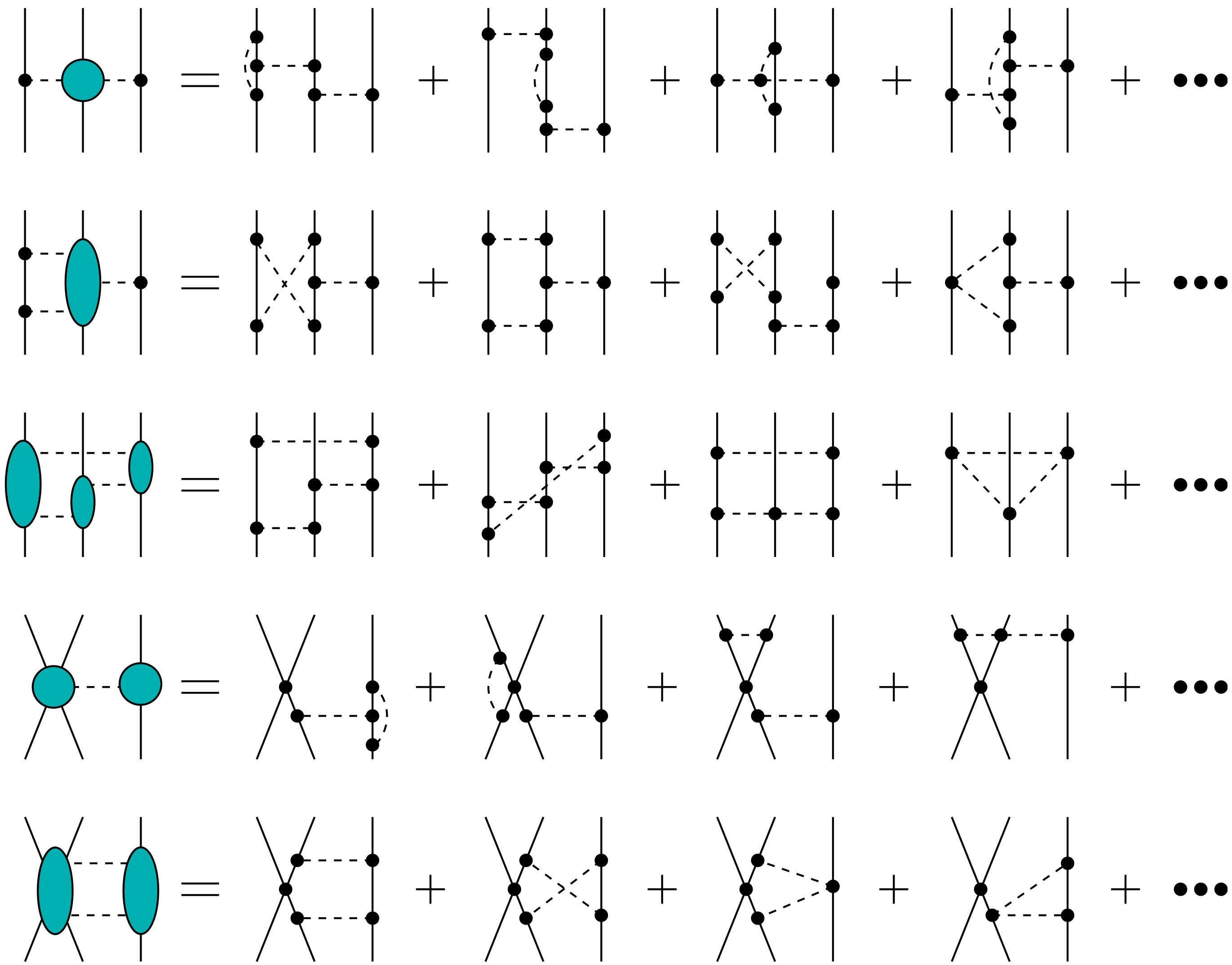
$$V_{3N}^{(3)} = \frac{g_A^2}{8F_\pi^4}\frac{\vec{\sigma}_1\cdot\vec{q}_1\vec{\sigma}_3\cdot\vec{q}_3}{[q_1^2+M_\pi^2][q_3^2+M_\pi^2]}[\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_3(-4c_1M_\pi^2$$
$$+2c_3\vec{q}_1\cdot\vec{q}_3)+c_4\boldsymbol{\tau}_1\times\boldsymbol{\tau}_3\cdot\boldsymbol{\tau}_2\vec{q}_1\times\vec{q}_3\cdot\vec{\sigma}_2]$$
$$-\frac{g_AD}{8F_\pi^2}\frac{\vec{\sigma}_3\cdot\vec{q}_3}{q_3^2+M_\pi^2}\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_3\vec{\sigma}_1\cdot\vec{q}_3+\frac{1}{2}E\boldsymbol{\tau}_2\cdot\boldsymbol{\tau}_3$$

Three undetermined πN couplings from NN fit



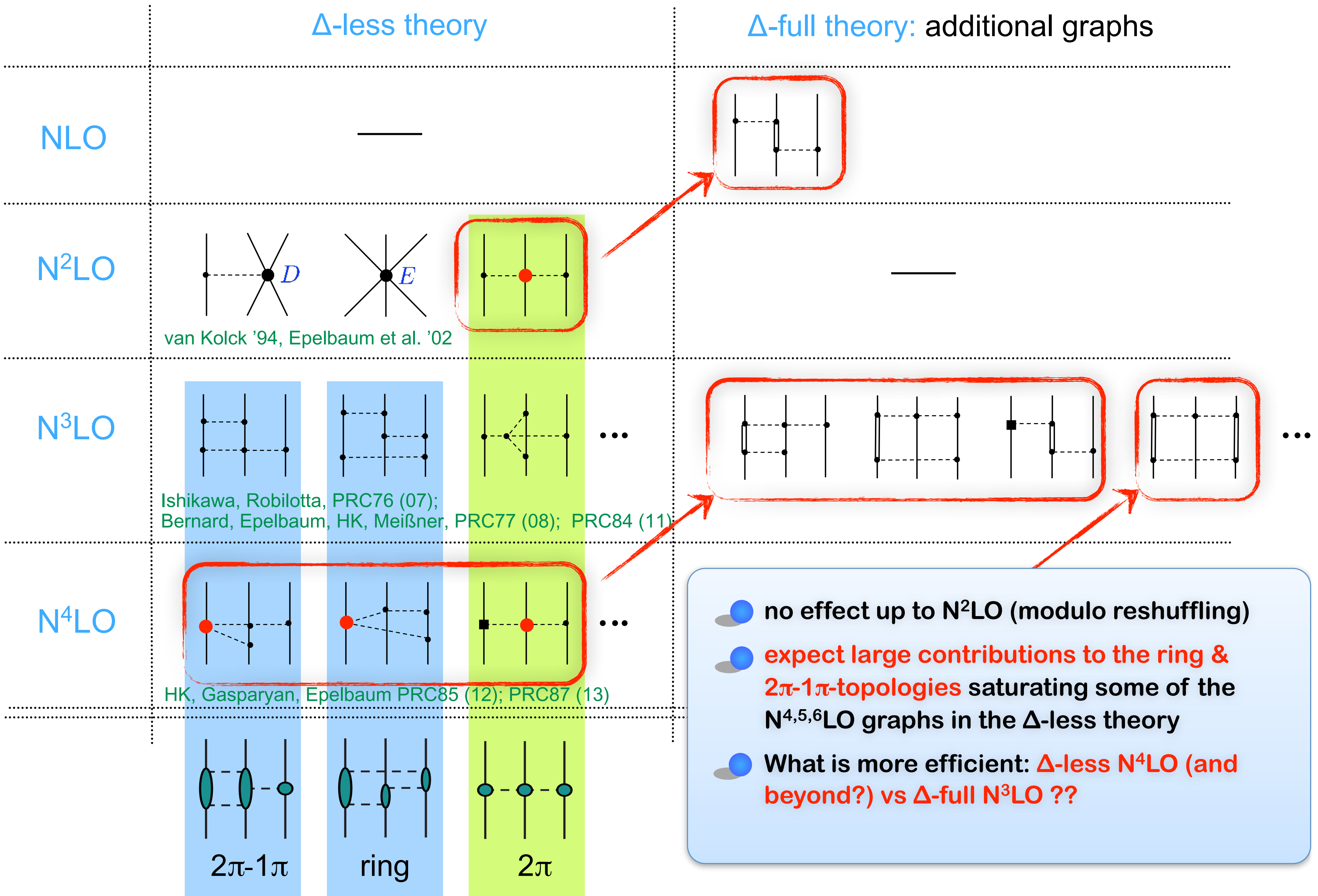
Chiral EFT: N^3LO 3N

Next-to-next-to-next-to-leading order $\nu = 4$



Good news: **no new constants** Bad news: well, there's all this

Aside: Effects of Adding Explicit Deltas



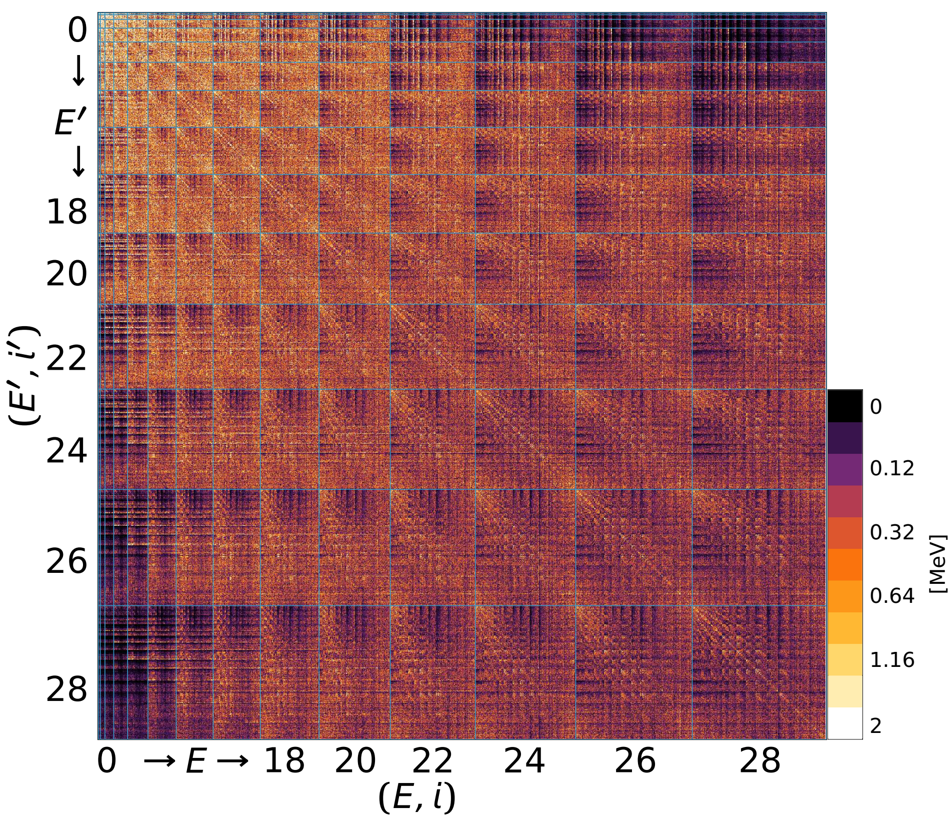
- no effect up to N^2LO (modulo reshuffling)
- expect large contributions to the ring & 2π-1π-topologies saturating some of the $N^{4,5,6}LO$ graphs in the Δ-less theory
- What is more efficient: Δ-less N^4LO (and beyond?) vs Δ-full N^3LO ??

Reshuffles effects to different chiral orders

SRG Evolution in HO Basis

Most common to SRG evolve 3N in HO basis:

3B-Jacobi HO matrix elements



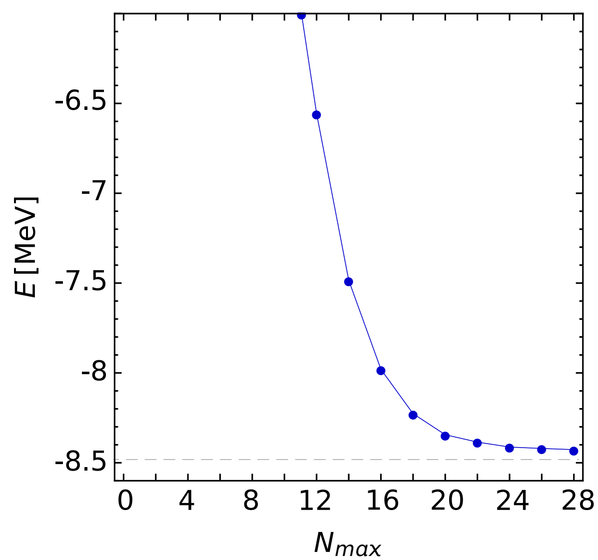
$$\alpha = 0.00\,\mathrm{fm}^4$$

$\lambda = \infty\,\mathrm{fm}^{-1}$

$$\langle E'i'JT|\,\widetilde{\mathrm{H}}_\alpha - \mathrm{T}_{\mathrm{int}}\,|EiJT\rangle$$

$$J^\pi = \tfrac{1}{2}^+, T = \tfrac{1}{2}, \hbar\Omega = 24\,\mathrm{MeV}$$

NCSM ground state ^{3}H



1) SRG-evolve both NN and 3N: NN+3N-full

2) NN Vlowk, refit 3N: NN+3N-fit

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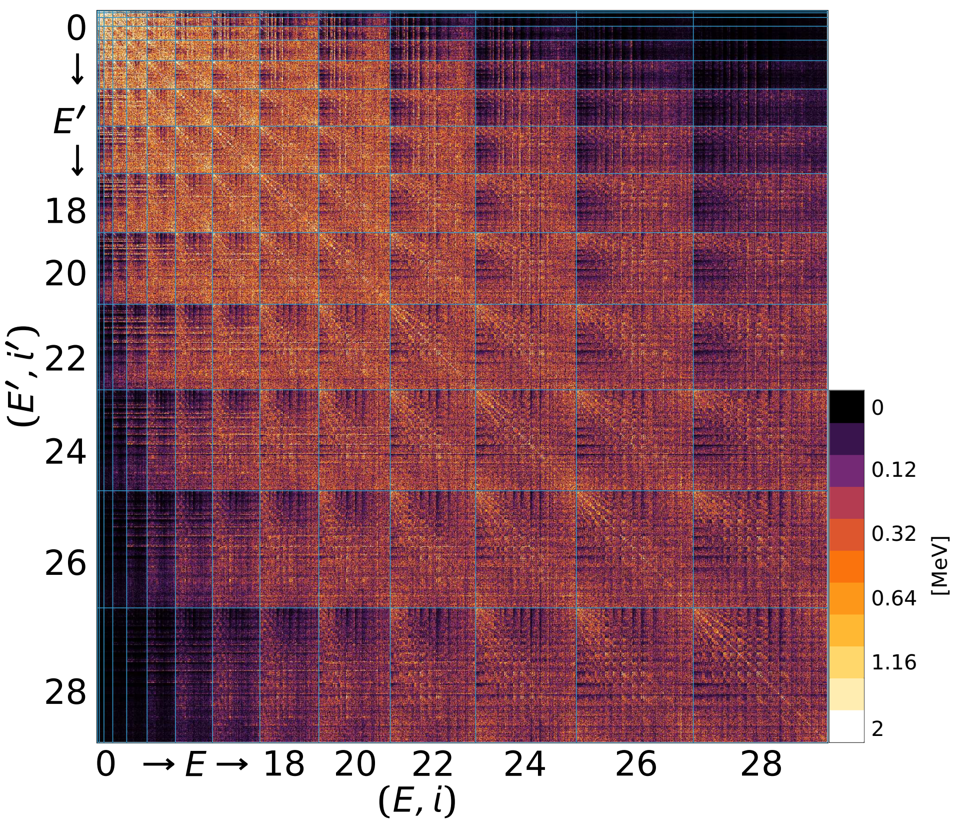


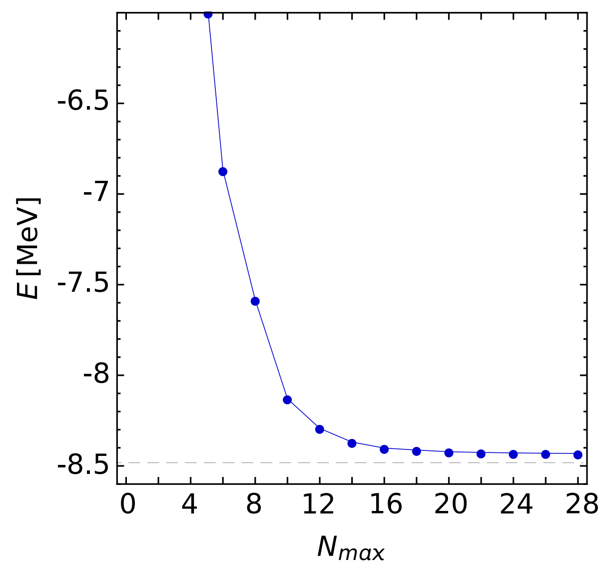
$$\alpha = 0.02\,\mathrm{fm}^4$$

$\lambda = 2.66\,\mathrm{fm}^{-1}$

$$\langle E' i' JT | \tilde{\mathrm{H}}_\alpha - \mathrm{T}_{\mathrm{int}} | E i J T \rangle$$

$$J^\pi = \tfrac{1}{2}^+, T = \tfrac{1}{2}, \hbar\Omega = 24\,\mathrm{MeV}$$

NCSM ground state ^{3}H



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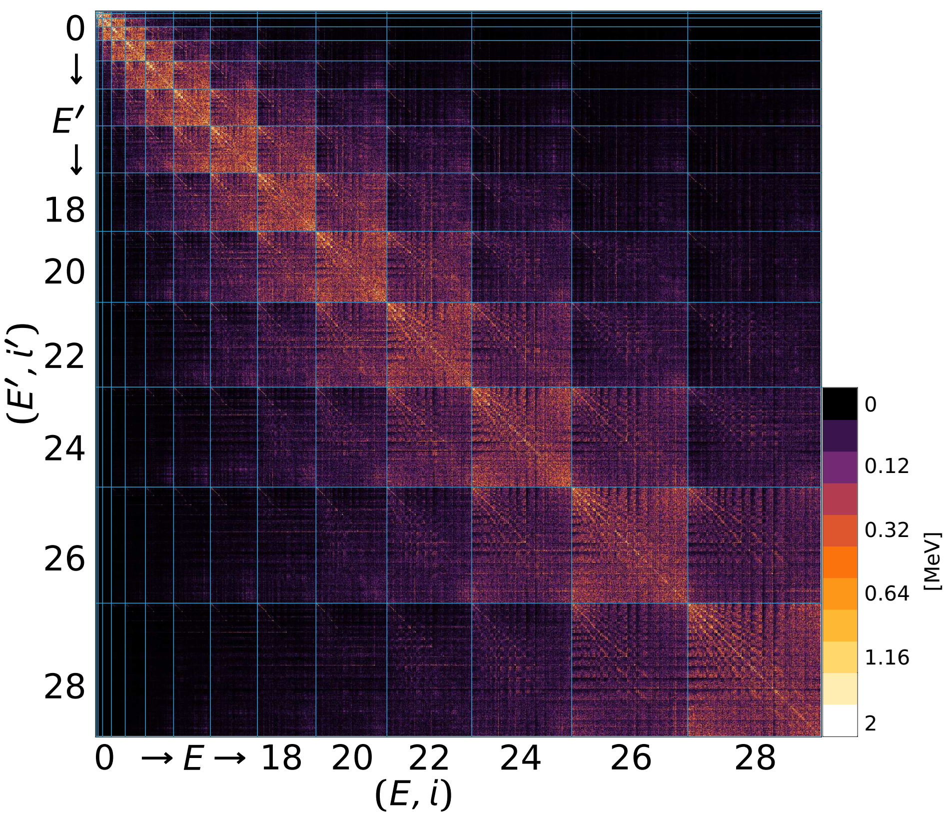


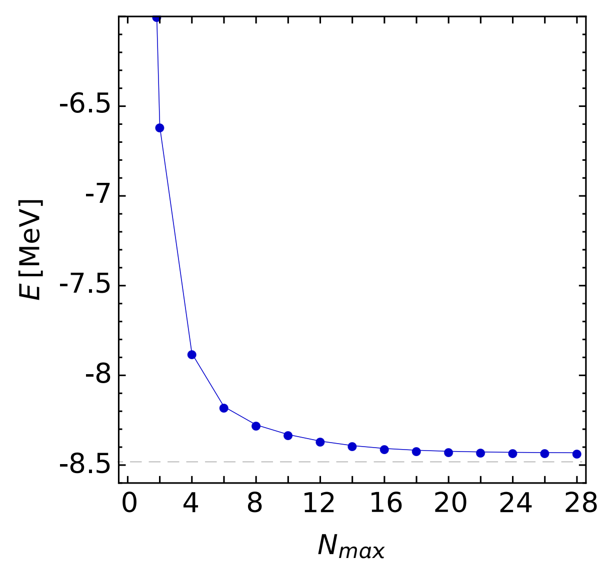
$\alpha = 1.28\,\mathrm{fm}^4$

$\lambda = 0.94\,\mathrm{fm}^{-1}$

$\langle E'i'JT|\,\widetilde{\mathrm{H}}_\alpha - \mathrm{T}_{\mathrm{int}}\,|EiJT\rangle$

$J^\pi = \frac{1}{2}^+, T = \frac{1}{2}, \hbar\Omega = 24\,\mathrm{MeV}$

NCSM ground state ^{3}H



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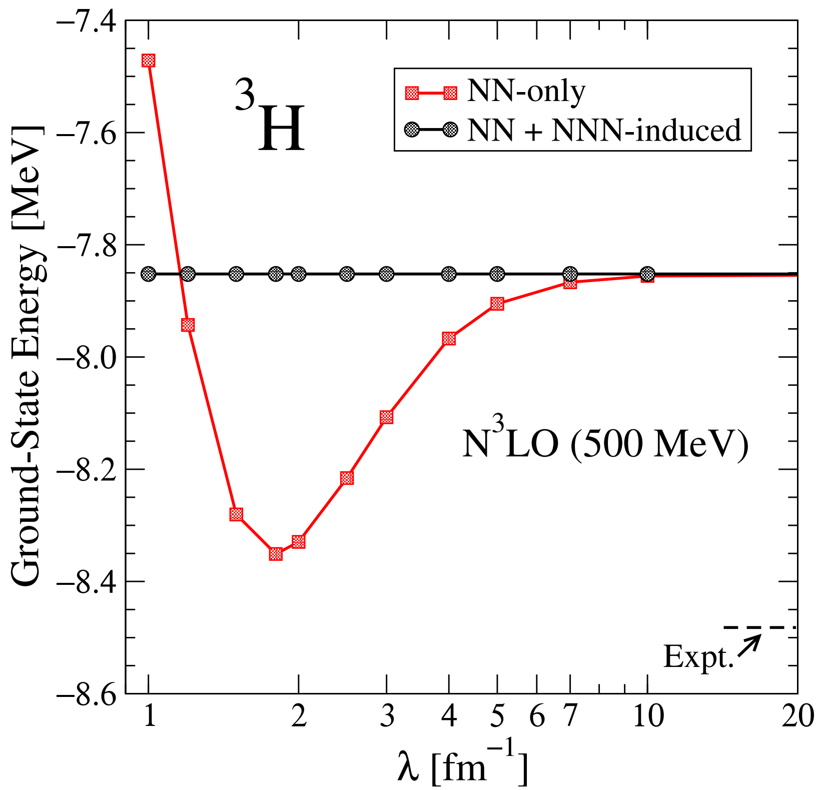
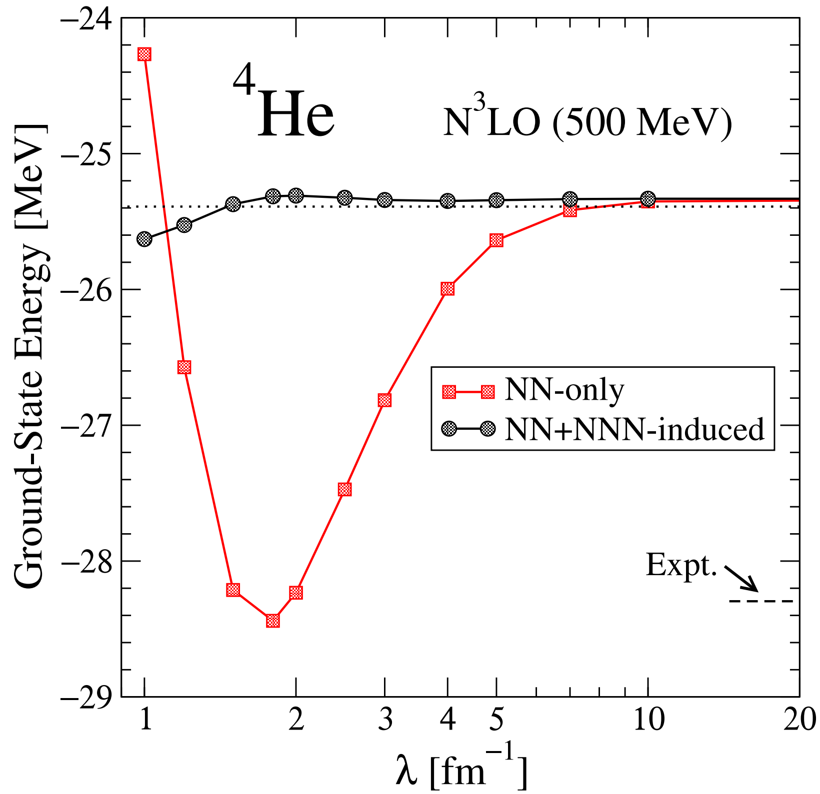
2) NN Vlowk, refit 3N: NN+3N-fit

Induced 3N Forces

Effect of including 3N-ind? Exactly initial V_{NN} up to neglected 4N-ind



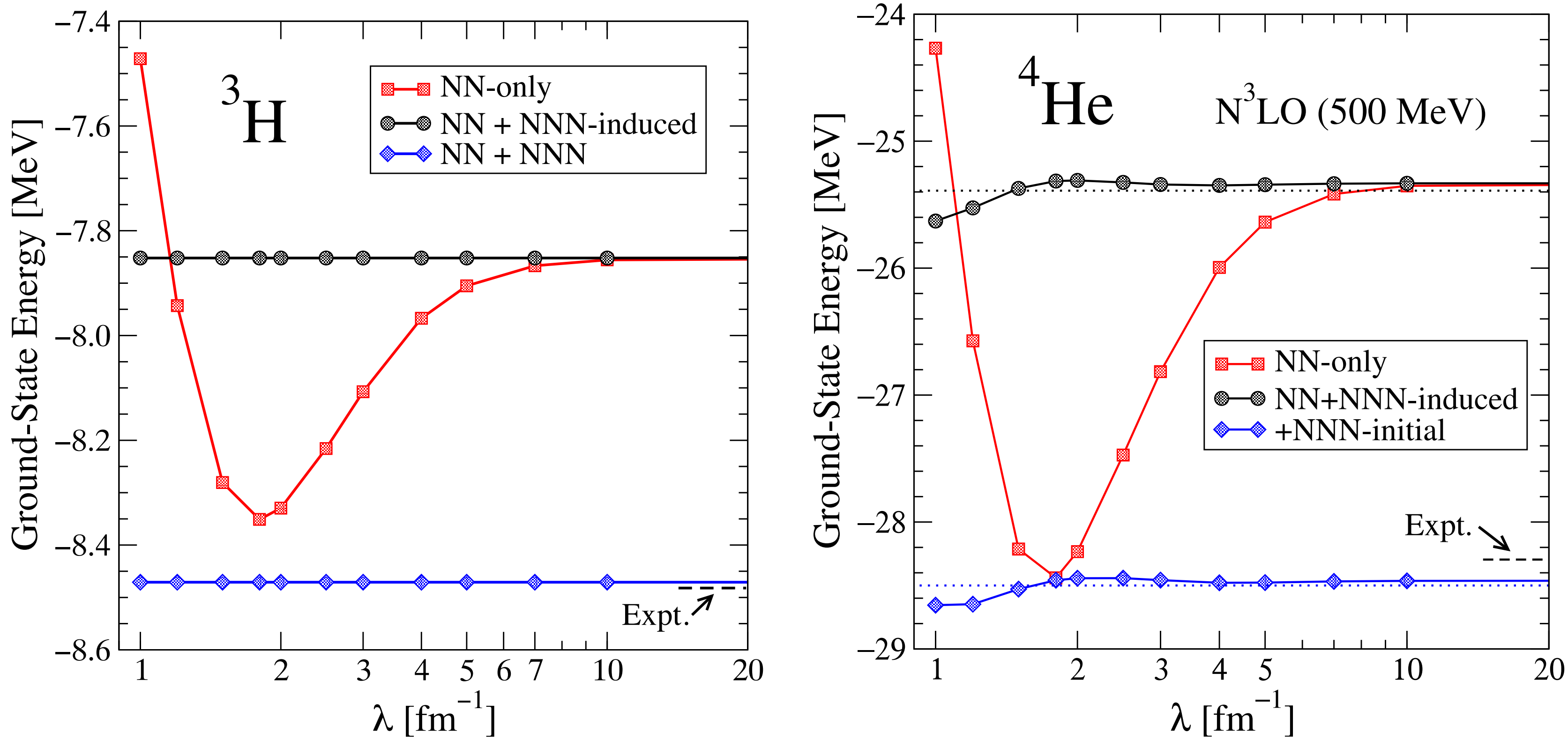


NN-only clear cutoff dependencs

3N-ind: dramatic reduction in cutoff dependence, no agreement with experiment

Induced 3N Forces

Effect of including 3N-ind? Exactly initial $V_{\rm NN}$ up to neglected 4N-ind



NN-only clear cutoff dependencs

3N-ind: dramatic reduction in cutoff dependence, no agreement with experiment

NN+3N-full retains cutoff independence, reproduces experiment!

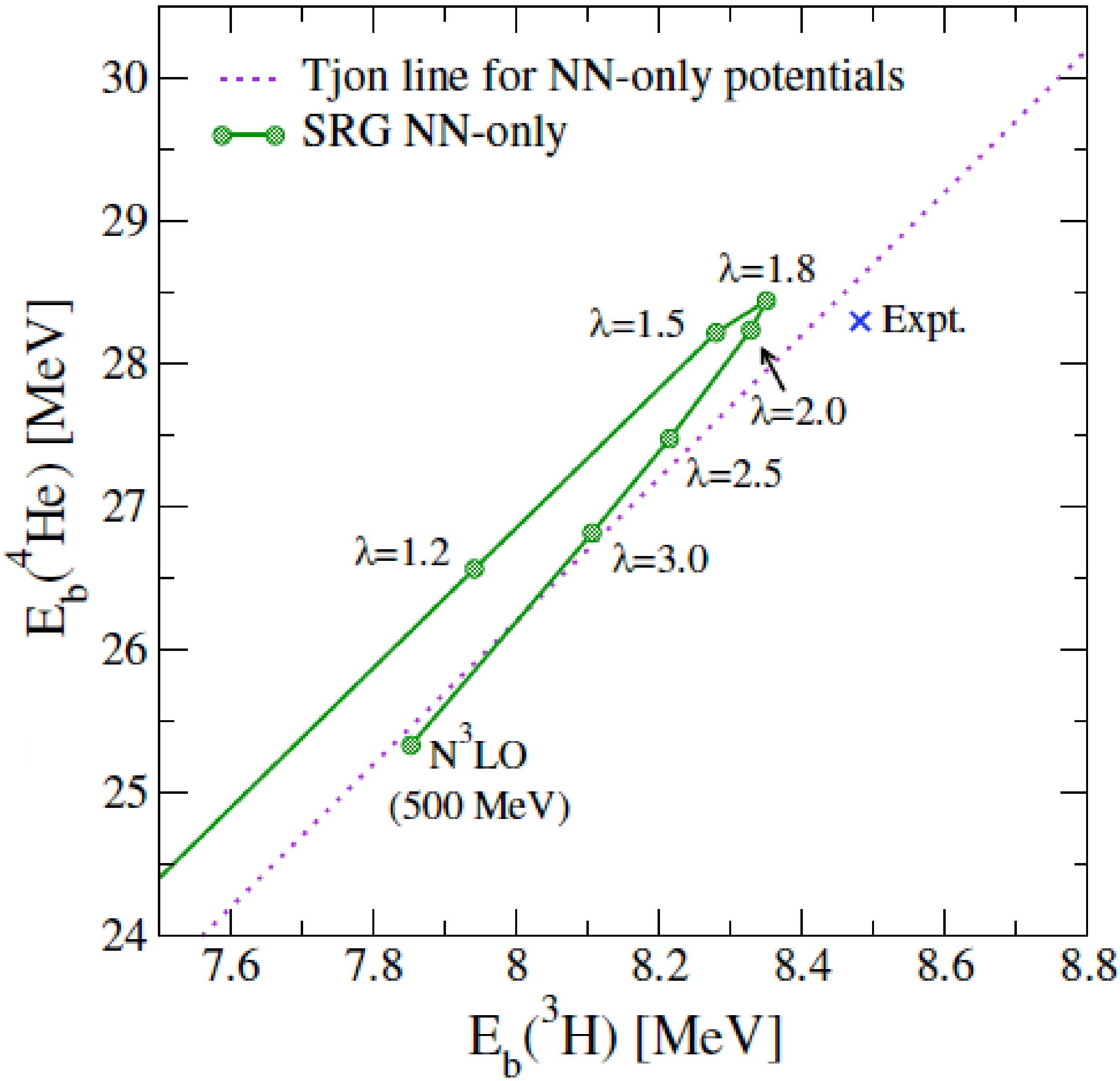
Benefits of Lower Cutoffs

Use cutoff dependence to assess missing physics: return to Tjon line

Varying cutoff moves along line

Still never reaches experiment

Tool, not a parameter!



Benefits of Lower Cutoffs

Use cutoff dependence to assess missing physics: return to Tjon line

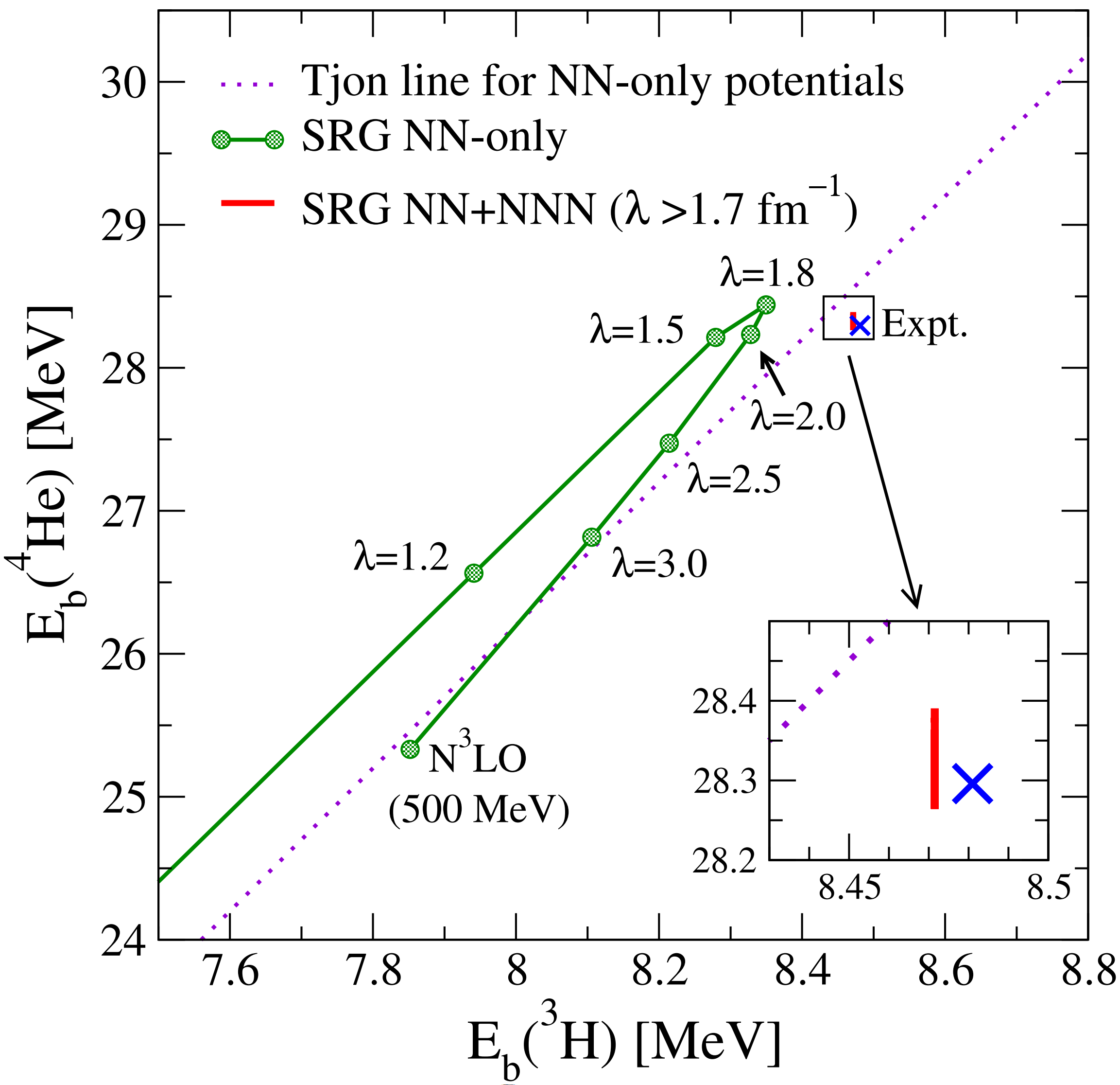
Varying cutoff moves along line

Still never reaches experiment

Tool, not a parameter!

Including 3N reaches expt.

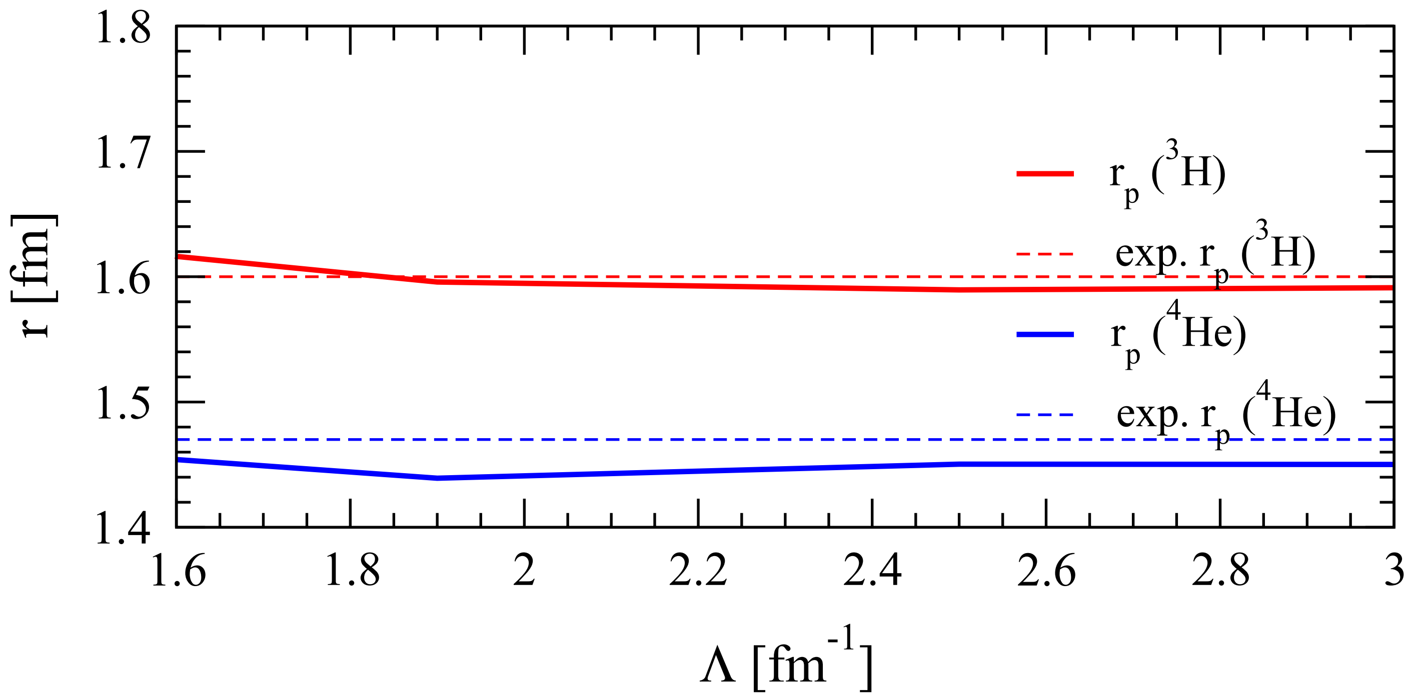
Why not perfect fit?



Cutoff Variation with 3N Forces

Use cutoff variation to assess missing physics in few body systems

Radii of triton and alpha particle calculated from NN+3N forces

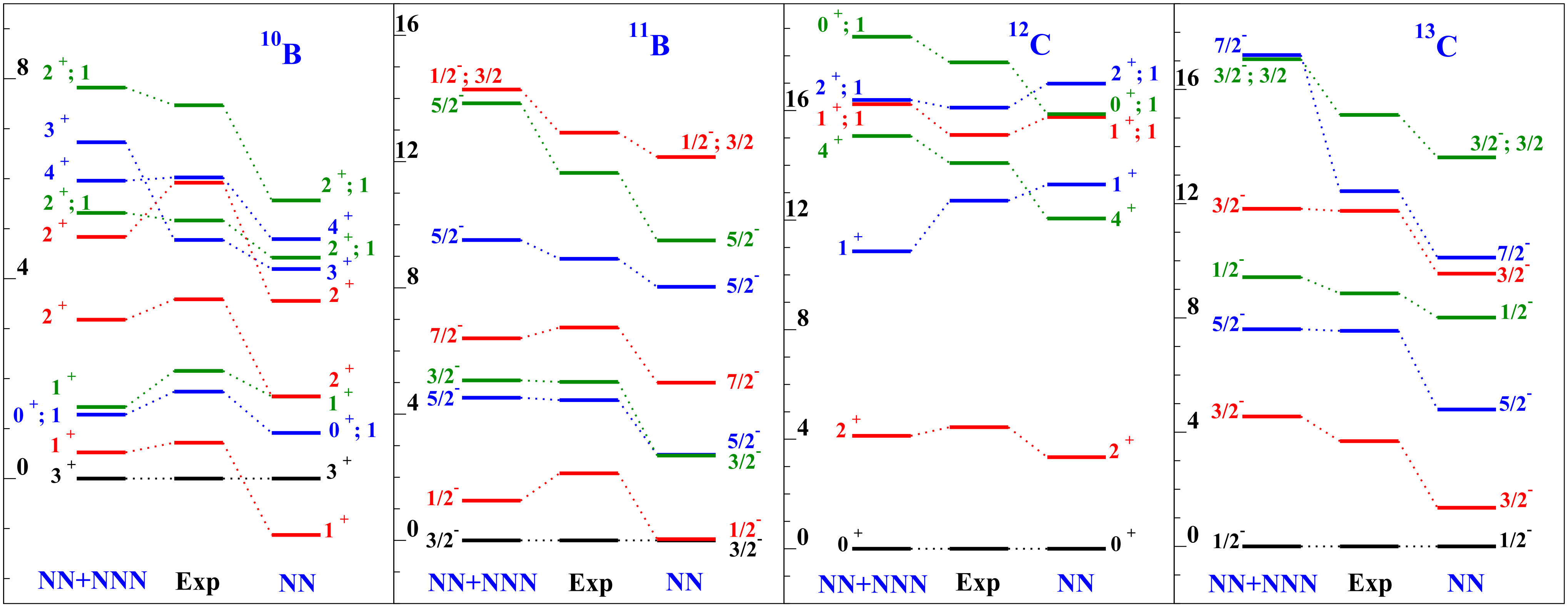


Minimal cutoff variation

Chiral Three-Body Forces in Light Nuclei

Importance of chiral 3N forces established in light nuclei

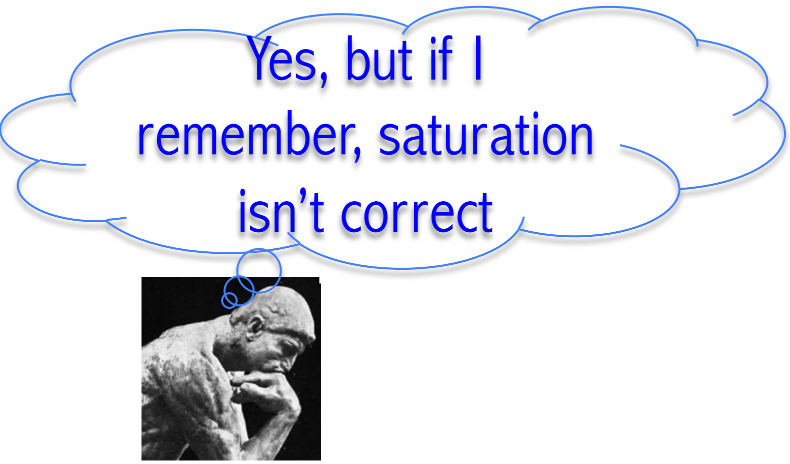
Converged NCSM (Navratil 2007)

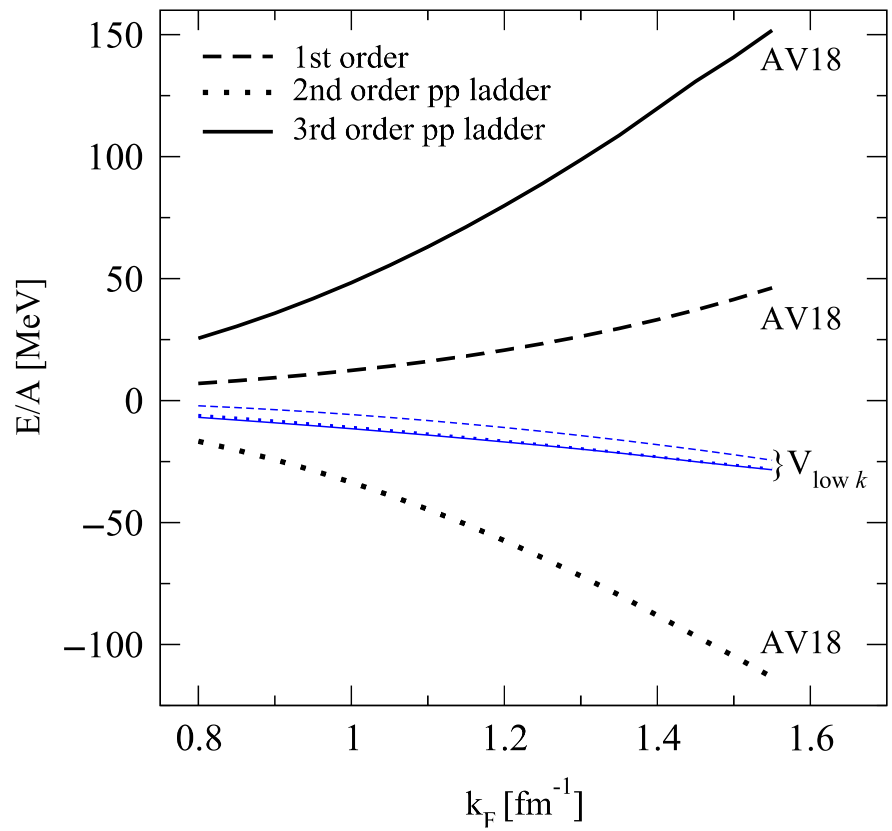


They work! What about nuclear matter?

Perturbative in Symmetric Nuclear Matter?

$$H(\Lambda) = T + V_{\mathrm{NN}}(\Lambda) + V_{3\mathrm{N}}(\Lambda) + V_{4\mathrm{N}}(\Lambda) + \cdots$$

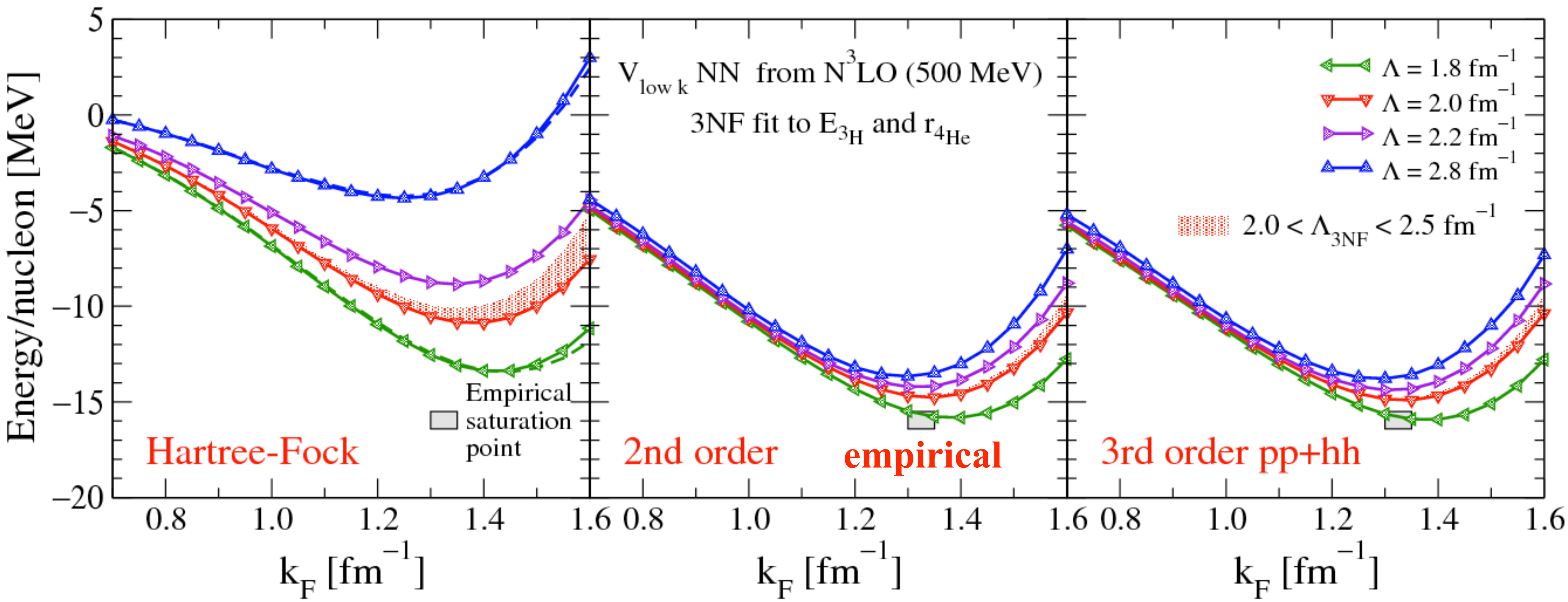




Significant improvement with low-momentum interactions!

Perturbative in Symmetric Nuclear Matter?

$$H(\Lambda) = T + V_{\rm NN}(\Lambda) + V_{\rm 3N}(\Lambda) + V_{\rm 4N}(\Lambda) + \cdots$$



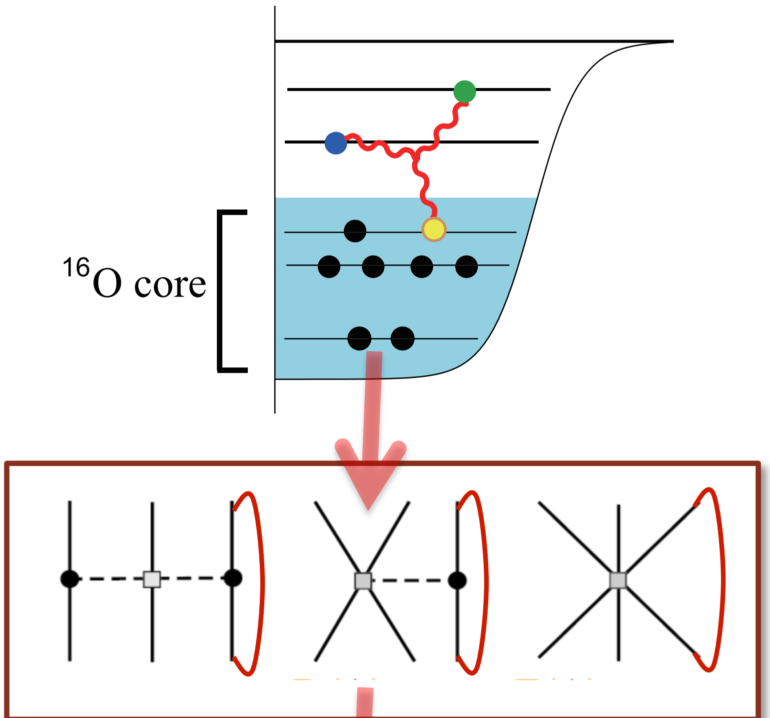
Now NN+3N-fit remain perturbative and reproduce saturation!

Minor but non-negligible cutoff variation

3N Forces for Valence-Shell Theories

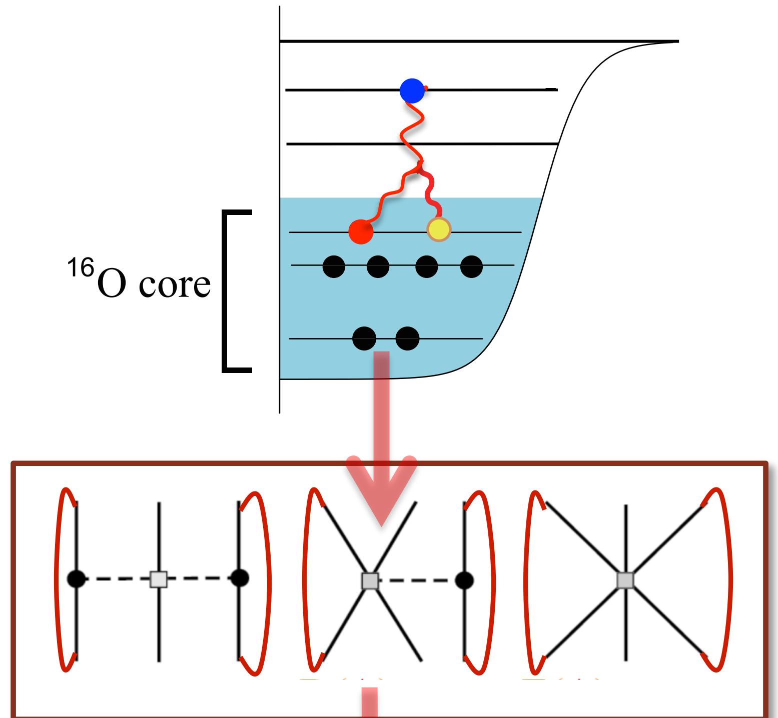
Normal-ordered 3N: contribution to valence neutron interactions

Effective two-body



$$\langle ab | V_{3N,\mathrm{eff}} | a'b' \rangle = \sum_{\alpha = core} \langle \alpha ab | V_{3N} | \alpha a'b' \rangle$$

Effective one-body



$$\langle a | V_{3N,\mathrm{eff}} | a' \rangle = \frac{1}{2} \sum_{\alpha\beta = core} \langle \alpha\beta a | V_{3N} | \alpha\beta a' \rangle$$

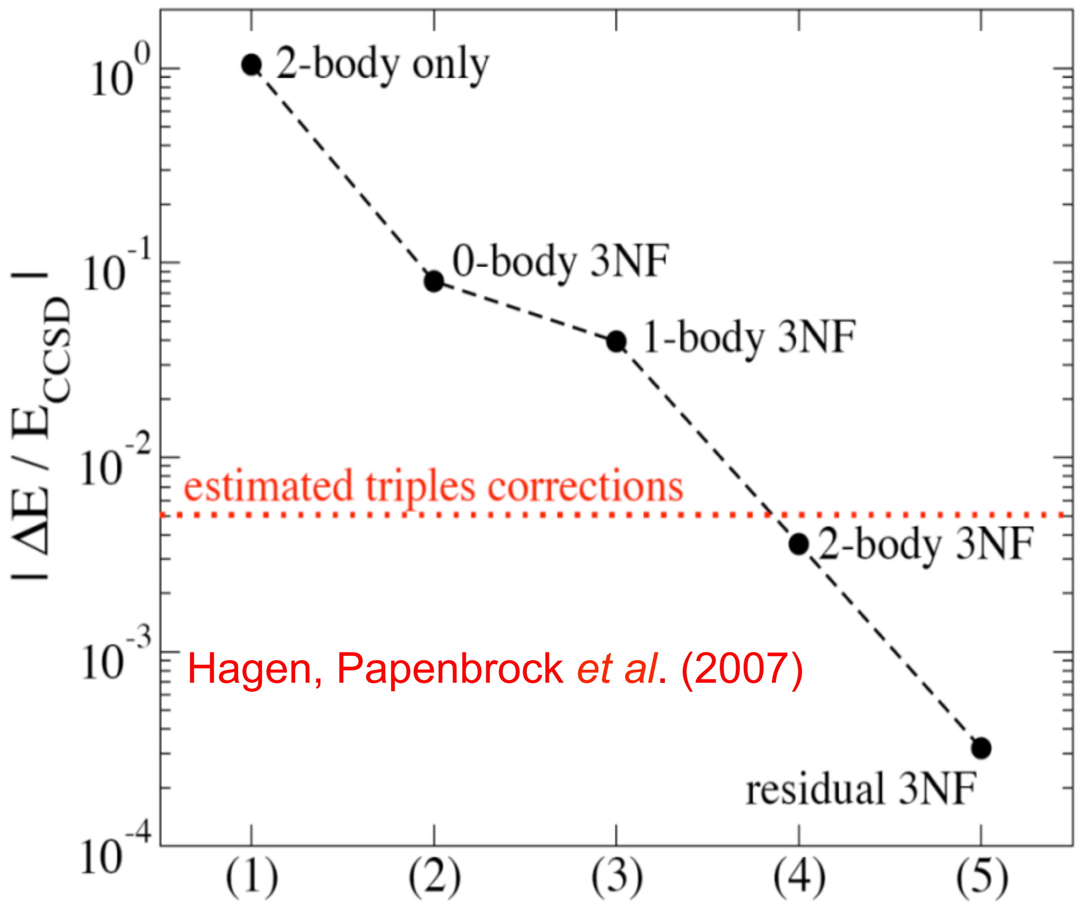
Combine with microscopic NN: eliminate empirical adjustments

3N Forces for Valence-Shell Theories

Effects of residual 3N between 3 valence nucleons?

Normal-ordered 3N: microscopic contributions to inputs for CI Hamiltonian

Effects of residual 3N between 3 valence nucleons?



Coupled-Cluster theory with 3N: benchmark of ^{4}He

0- 1- and 2-body of 3NF dominate

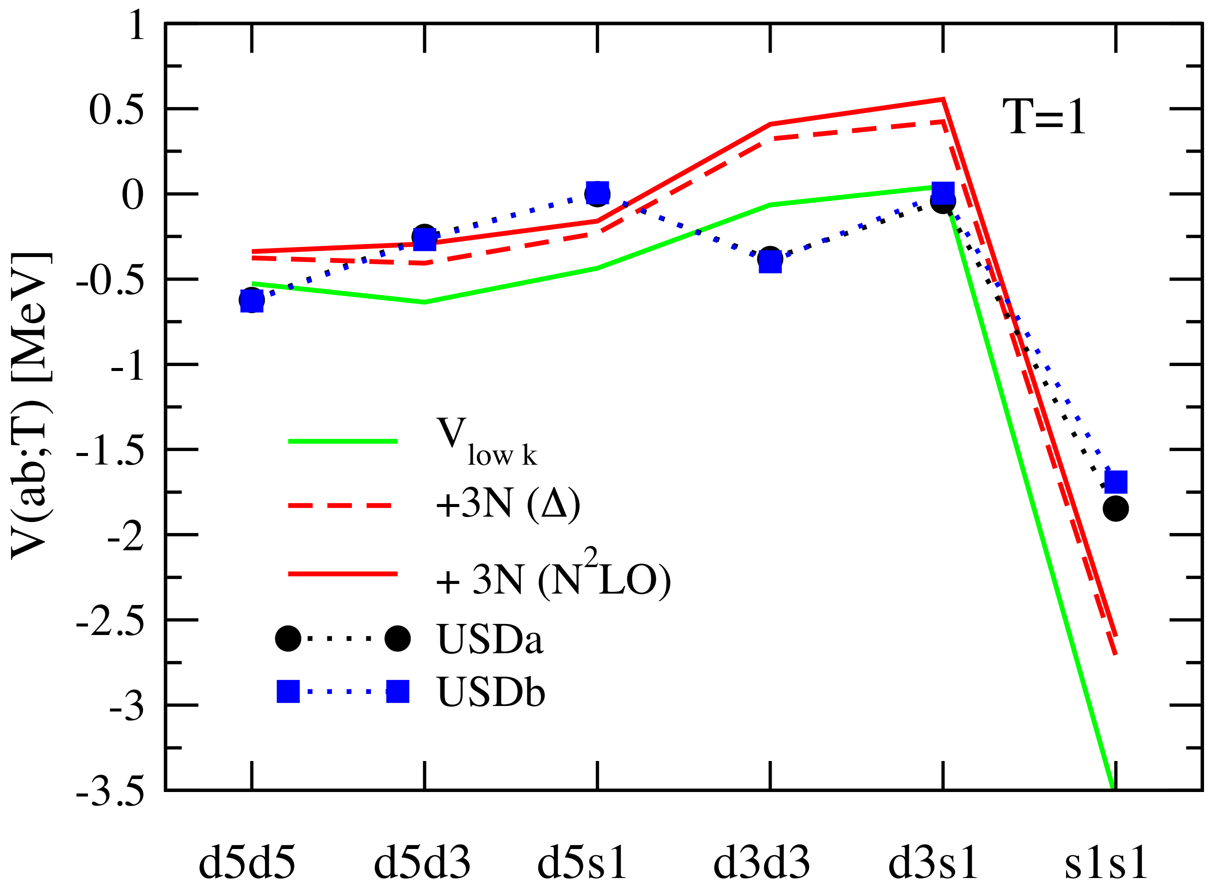
Residual 3N can be neglected

Work on ^{16}O in progress

Approximated residual 3N by summing over valence nucleon

– Nucleus-dependent: effect small, not negligible by ^{24}O

Two-body 3N: Monopoles in *sd*-shell



Dominant effect from **one-Δ** – as expected from cutoff variation

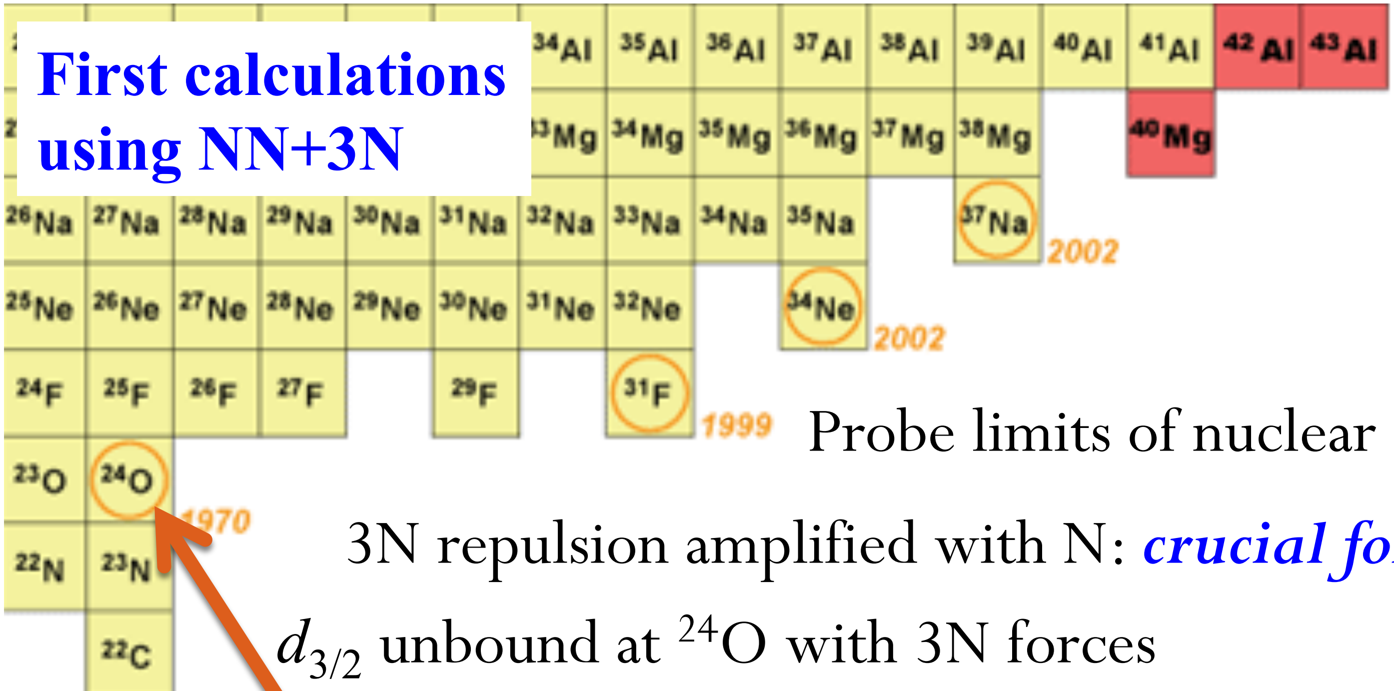
3N forces produce clear repulsive shift in monopoles

First calculations to show missing monopole strength due to neglected 3N

Future: Improved treatment of high-lying orbits

Oxygen Anomaly

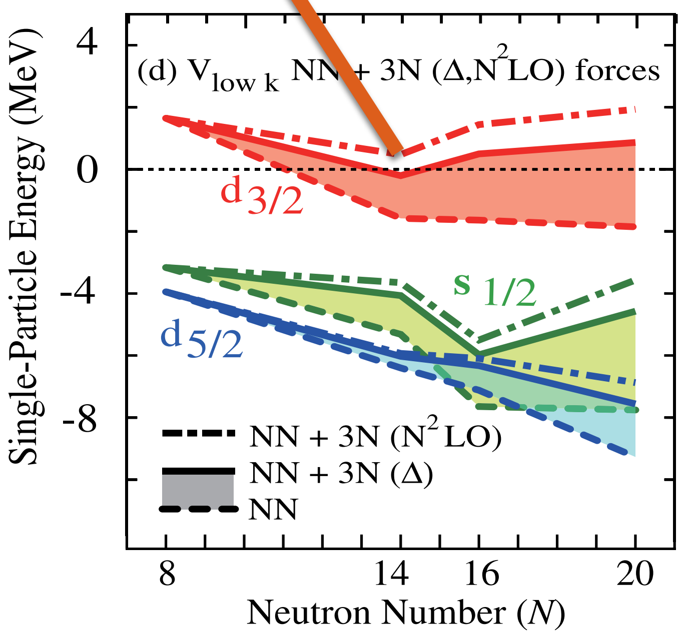
First calculations using NN+3N



Probe limits of nuclear existence with 3N forces

3N repulsion amplified with N: ***crucial for neutron-rich nuclei***

$d_{3/2}$ unbound at ^{24}O with 3N forces



Otsuka, Suzuki, JDH, Schwenk, Akaishi, PRL (2010)

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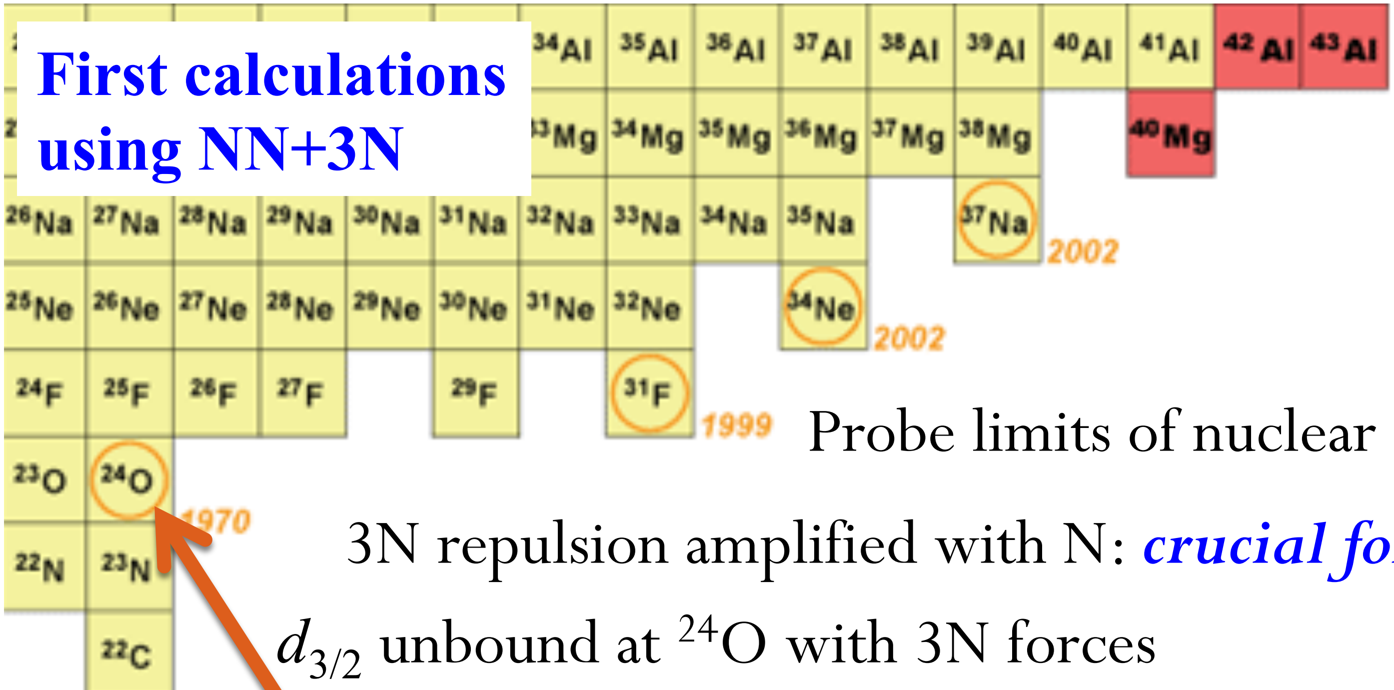
First calculations using NN+3N



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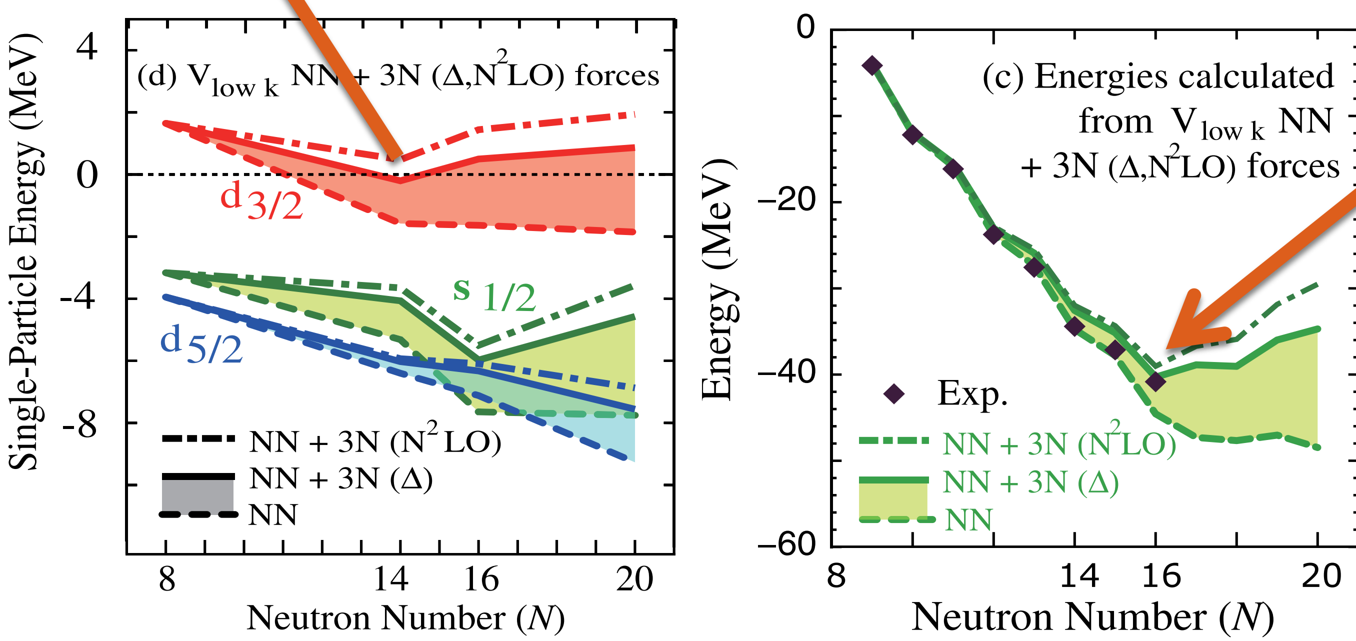


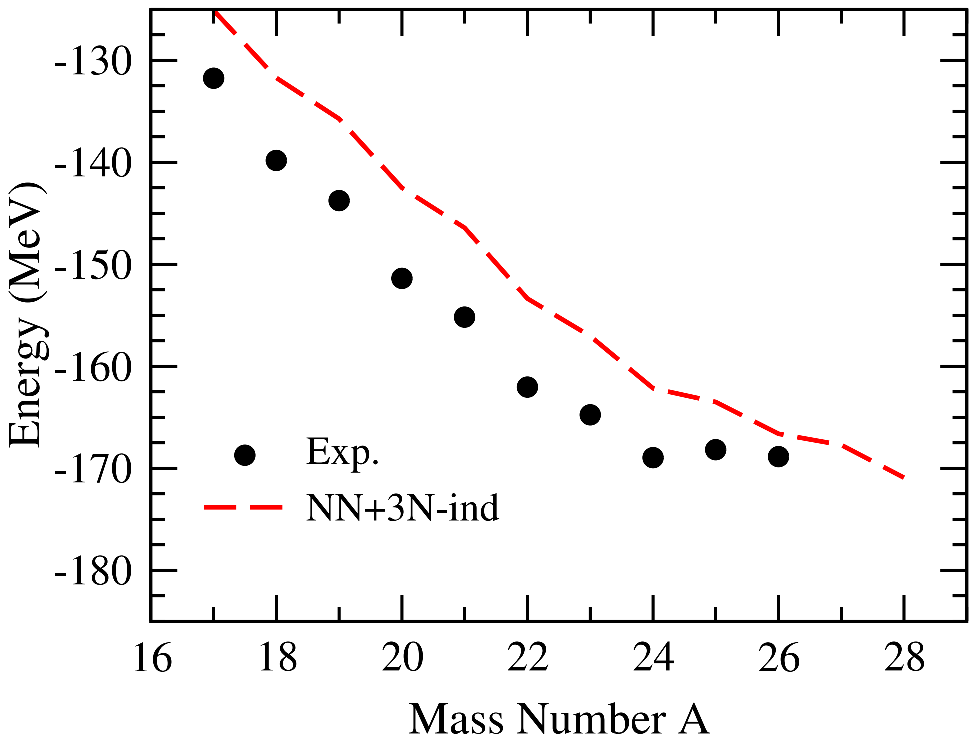
Isotopes unbound beyond ^{24}O

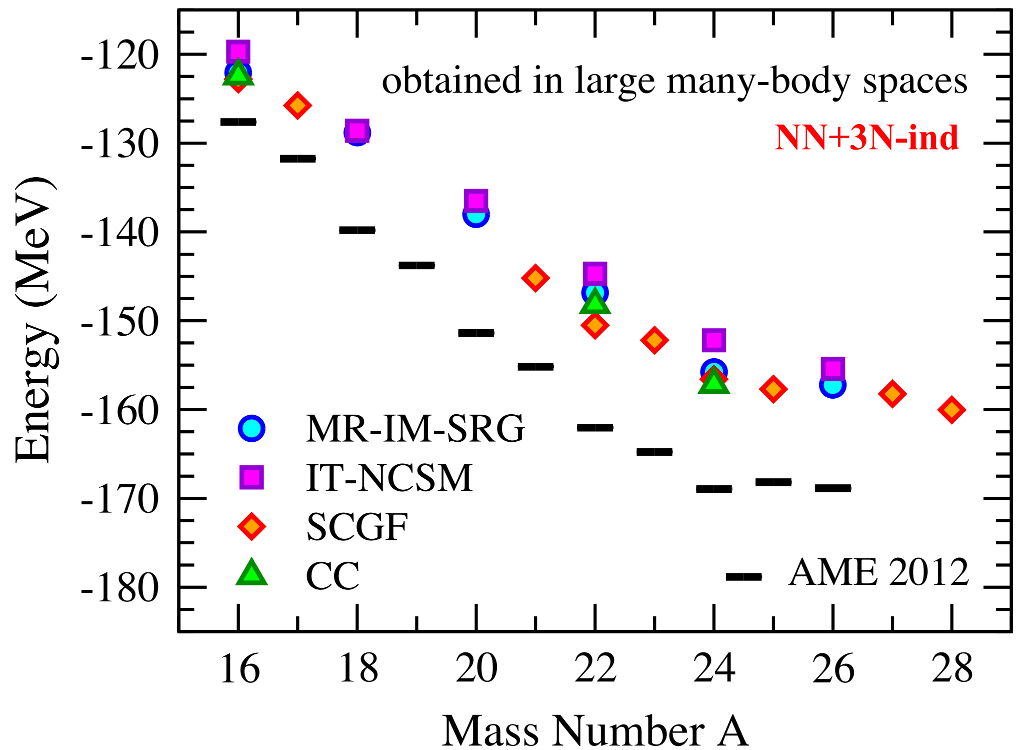
First microscopic explanation of oxygen anomaly

Otsuka, Suzuki, JDH, Schwenk, Akaishi, PRL (2010)

Comparison with Large-Space Methods

Large-space methods with **same SRG-evolved NN+3N-ind forces**





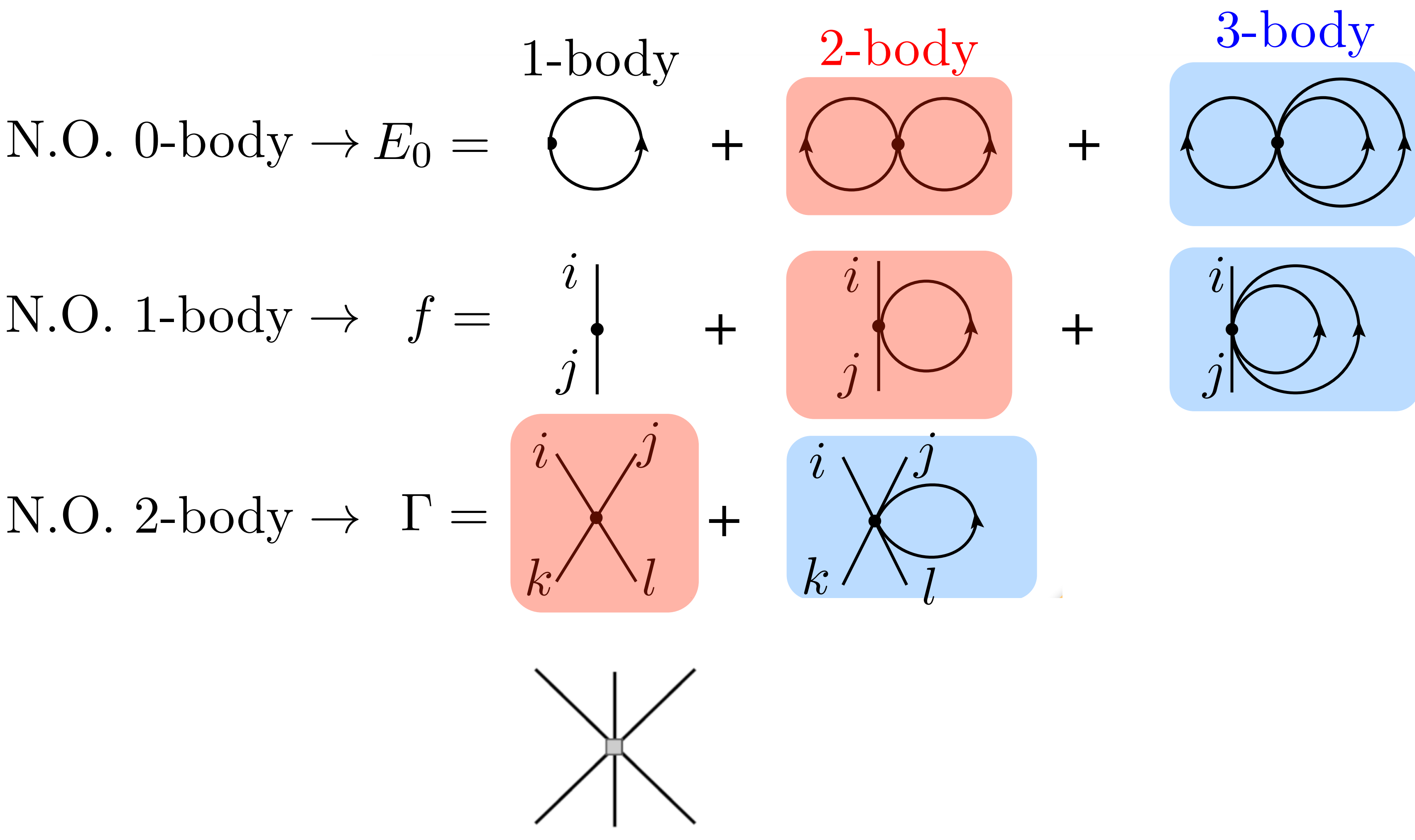
Agreement between all methods with same input forces

No reproduction of dripline in any case

Normal-Ordered Hamiltonian

Now rewrite exactly the initial Hamiltonian in normal-ordered form

$$H_{\mathrm{N.O.}} = E_0 + \sum_{ij} f_{ij} \left\{a_i^\dagger a_j\right\} + \frac{1}{4}\sum_{jkl} \Gamma_{ijkl} \left\{a_i^\dagger a_j^\dagger a_l a_k\right\} + \frac{1}{36} \sum_{ijklmn} W_{ijklmn} \left\{a_i^\dagger a_j^\dagger a_k^\dagger a_l a_m a_n\right\}$$



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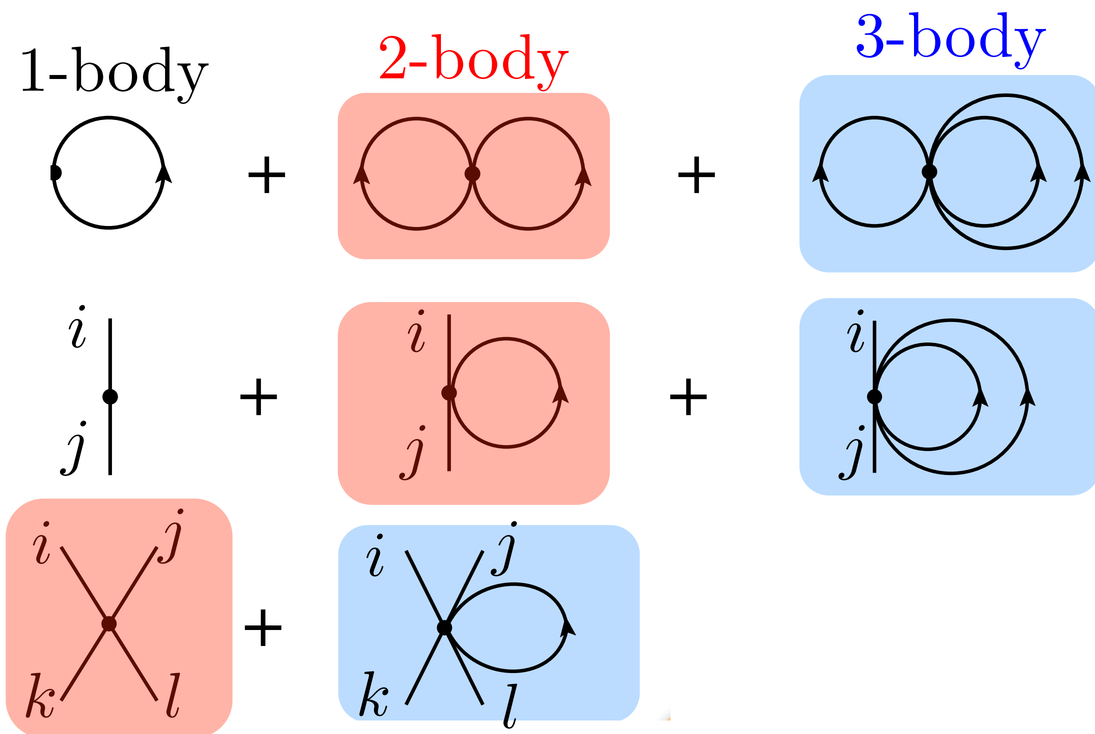
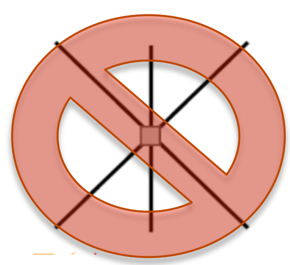
N.O. 0-body $\rightarrow E_0 =$

N.O. 1-body $\rightarrow \quad f =$

N.O. 2-body $\rightarrow \quad \Gamma =$



Neglect residual 3N



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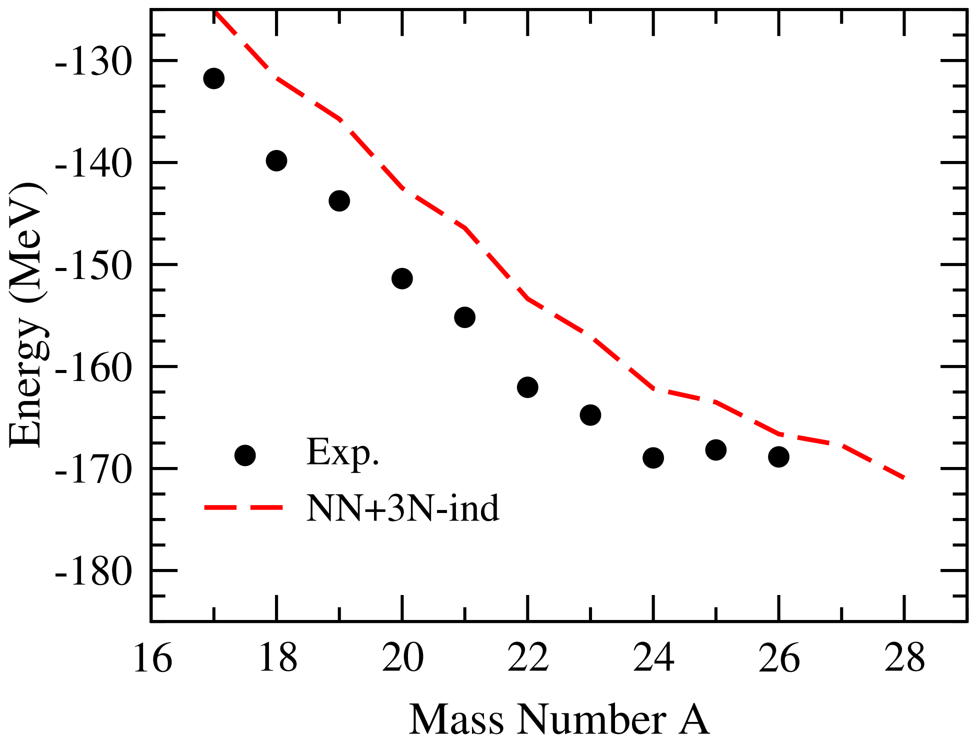
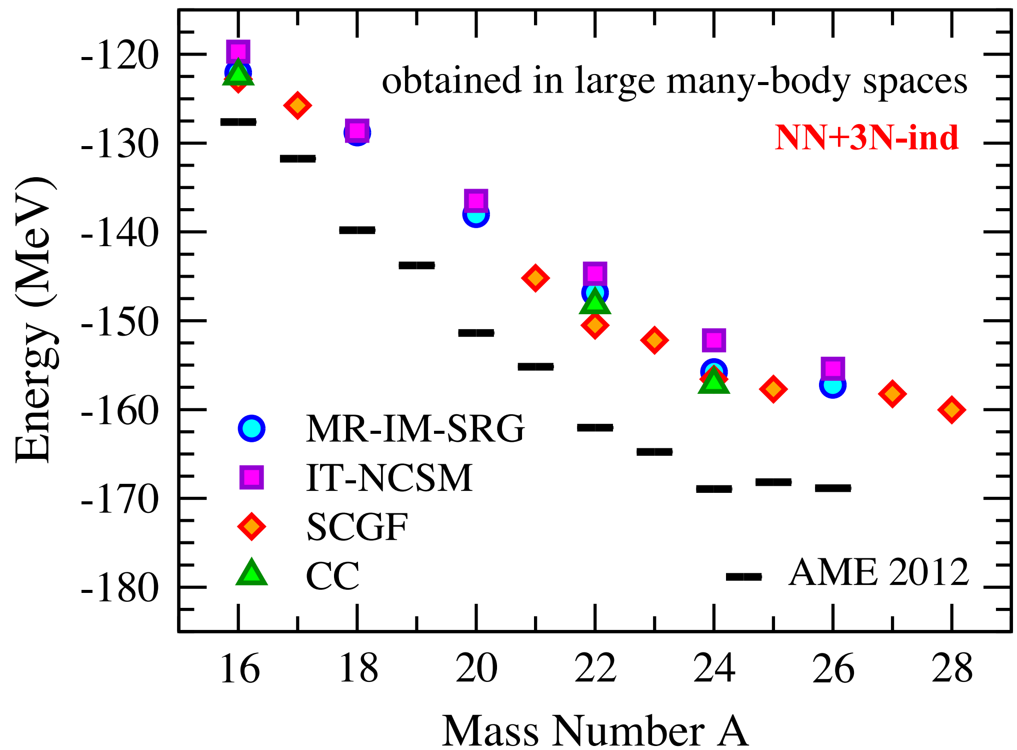
Large-space methods with **same SRG-evolved NN+3N-ind forces**





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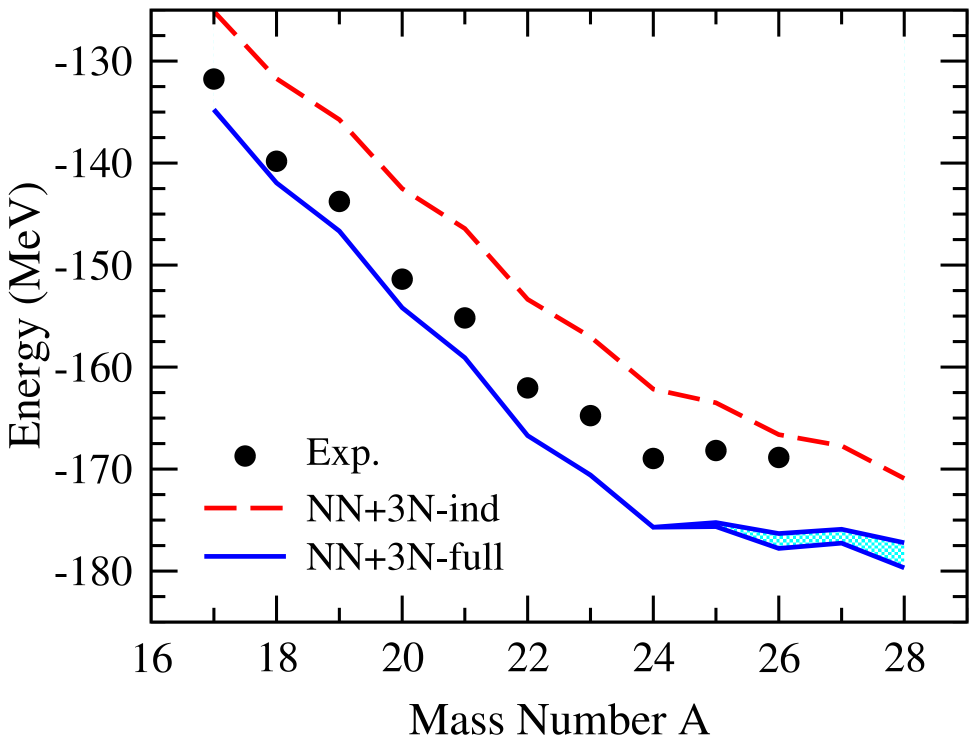
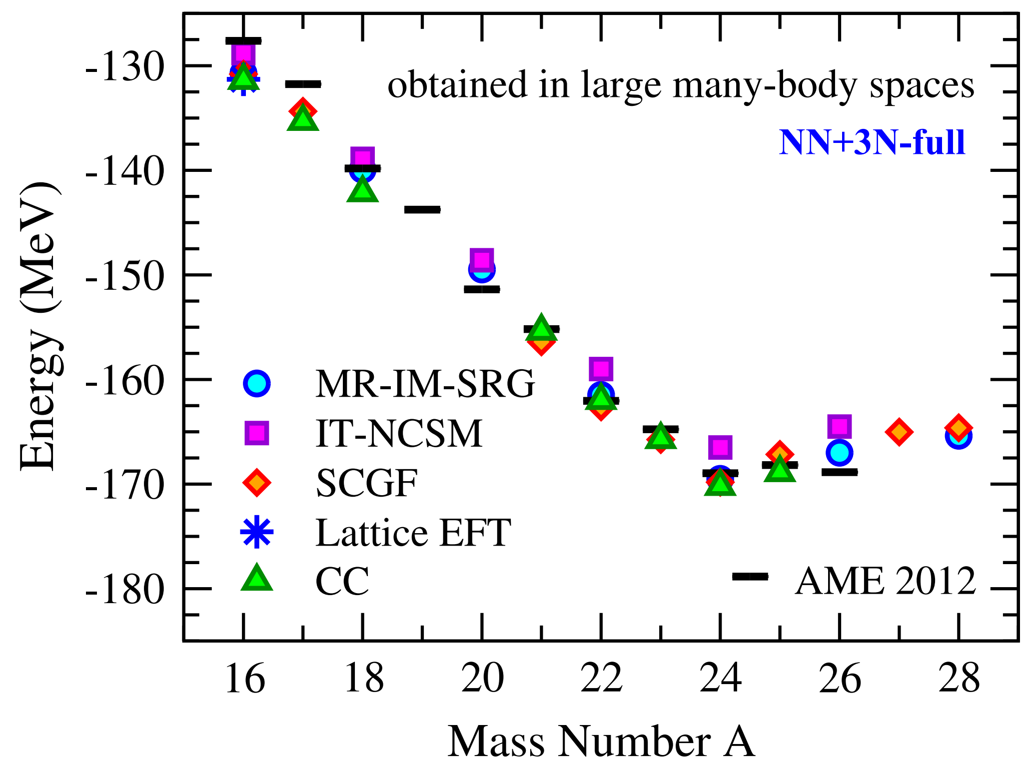
Comparison with Large-Space Methods

Large-space methods with **same SRG-evolved NN+3N-full forces**





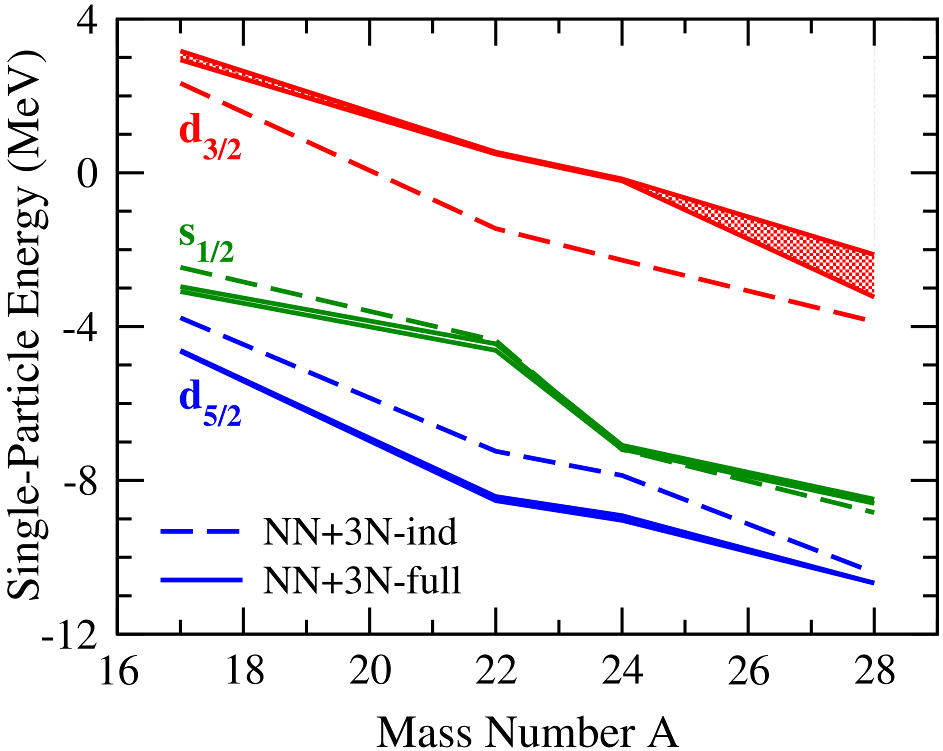
Agreement between all methods with same input forces

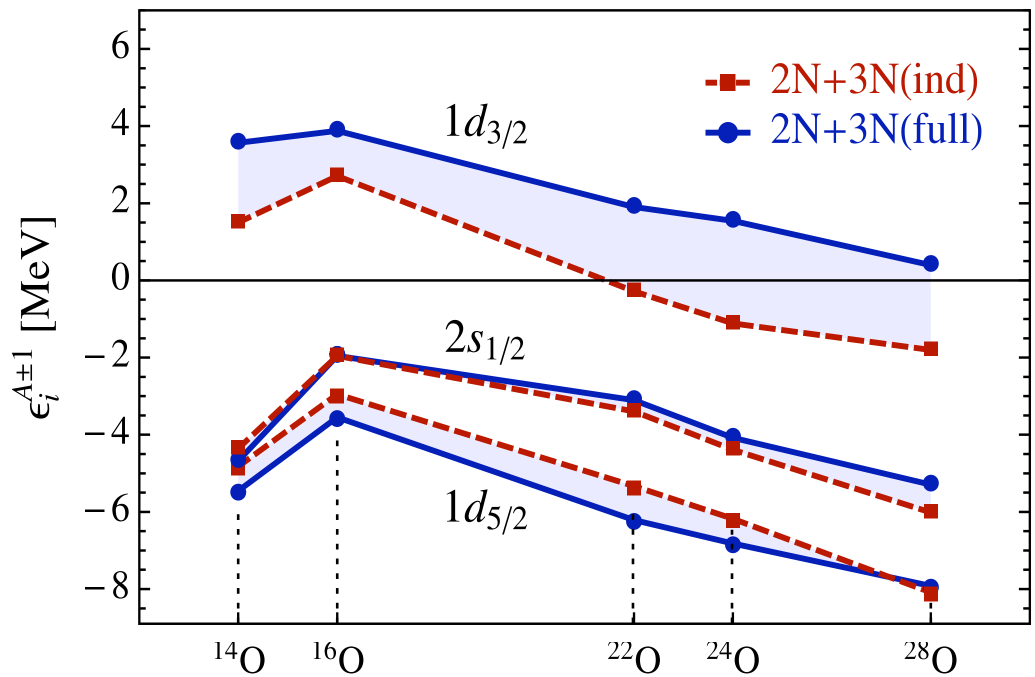
Clear improvement with NN+3N-full

Validates valence-space results

Oxygen Dripline Mechanism

Self-consistent Green's Function with **same SRG-evolved NN+3N forces**





Cipollone, Barbieri, Navrátil, PRL (2013)

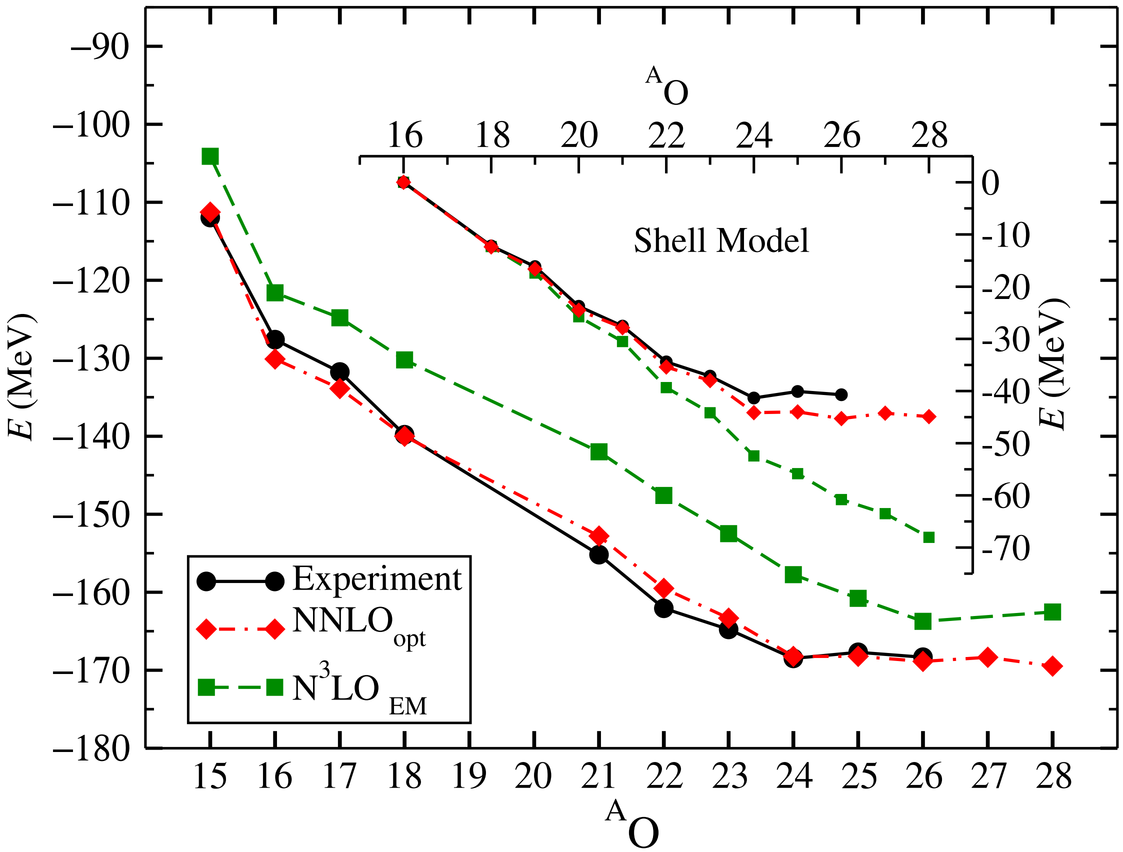
Robust mechanism driving dripline behavior

3N repulsion raises $d_{3/2}$, lessens decrease across shell

Similar to first MBPT NN+3N calculations in oxygen

Optimized Chiral Forces N²LO NN-Only

Recent calculations at N^{2}LO without 3N forces found a remarkable result



Ekström et al (PRL 2013)

Oxygen dripline reproduced with NN forces only!

What does this mean about 3N?

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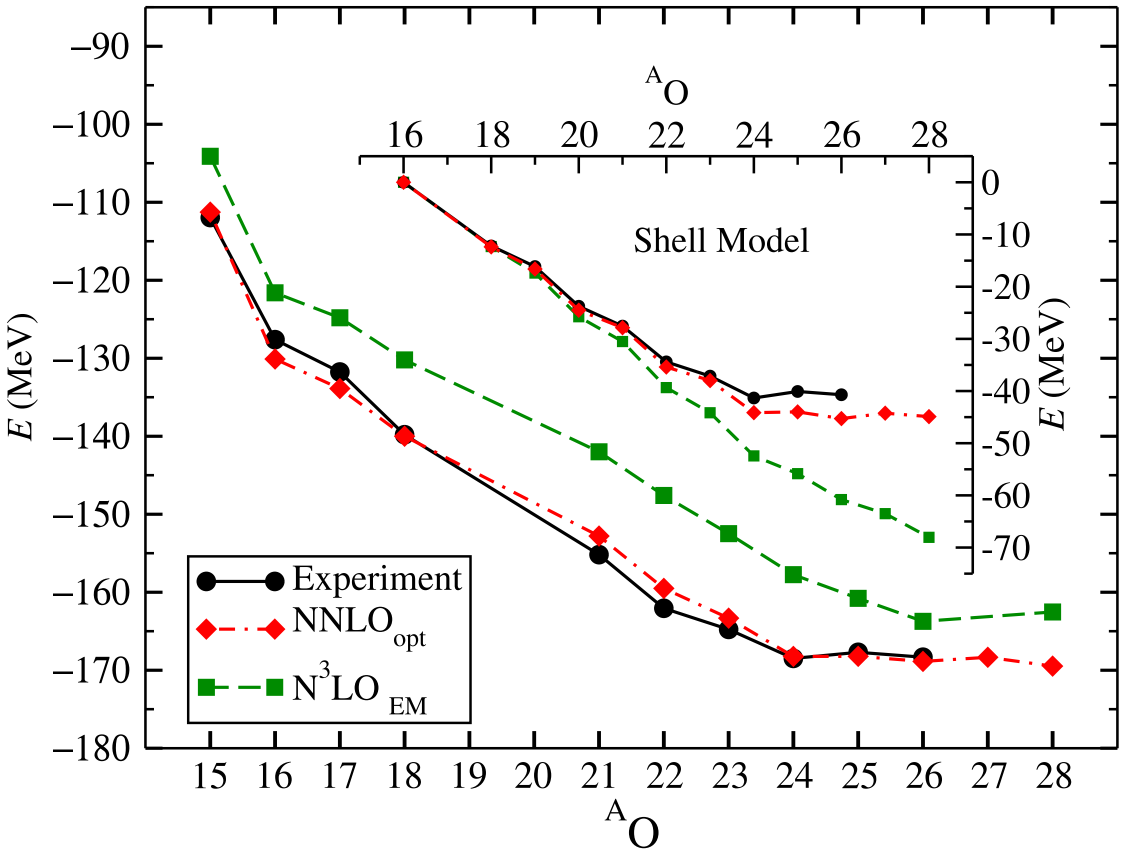
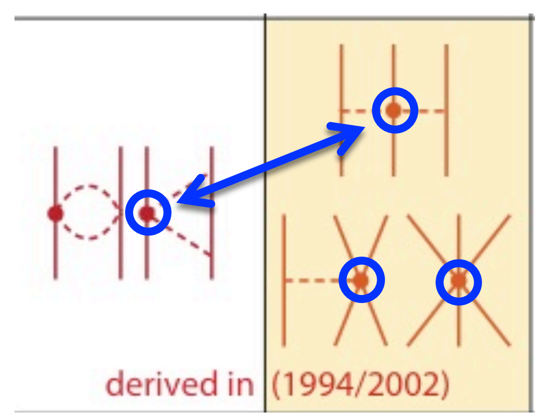


Ekström et al (PRL 2013)

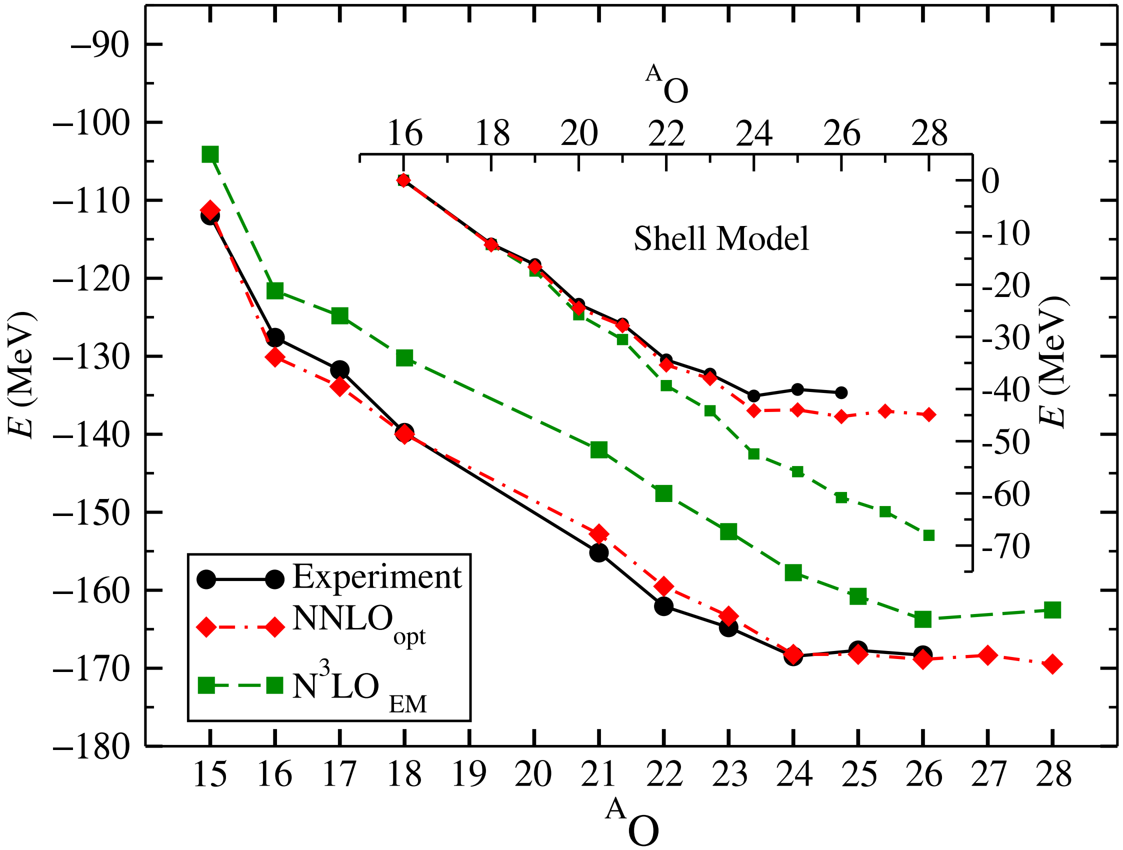
Oxygen dripline reproduced with NN forces only!

Power counting dictates 3N forces be included



Optimized Chiral Forces N^{2}LO NN-Only

Recent calculations at N^{2}LO without 3N forces found a remarkable result



Ekström et al (PRL 2013)

Oxygen dripline reproduced with NN forces only

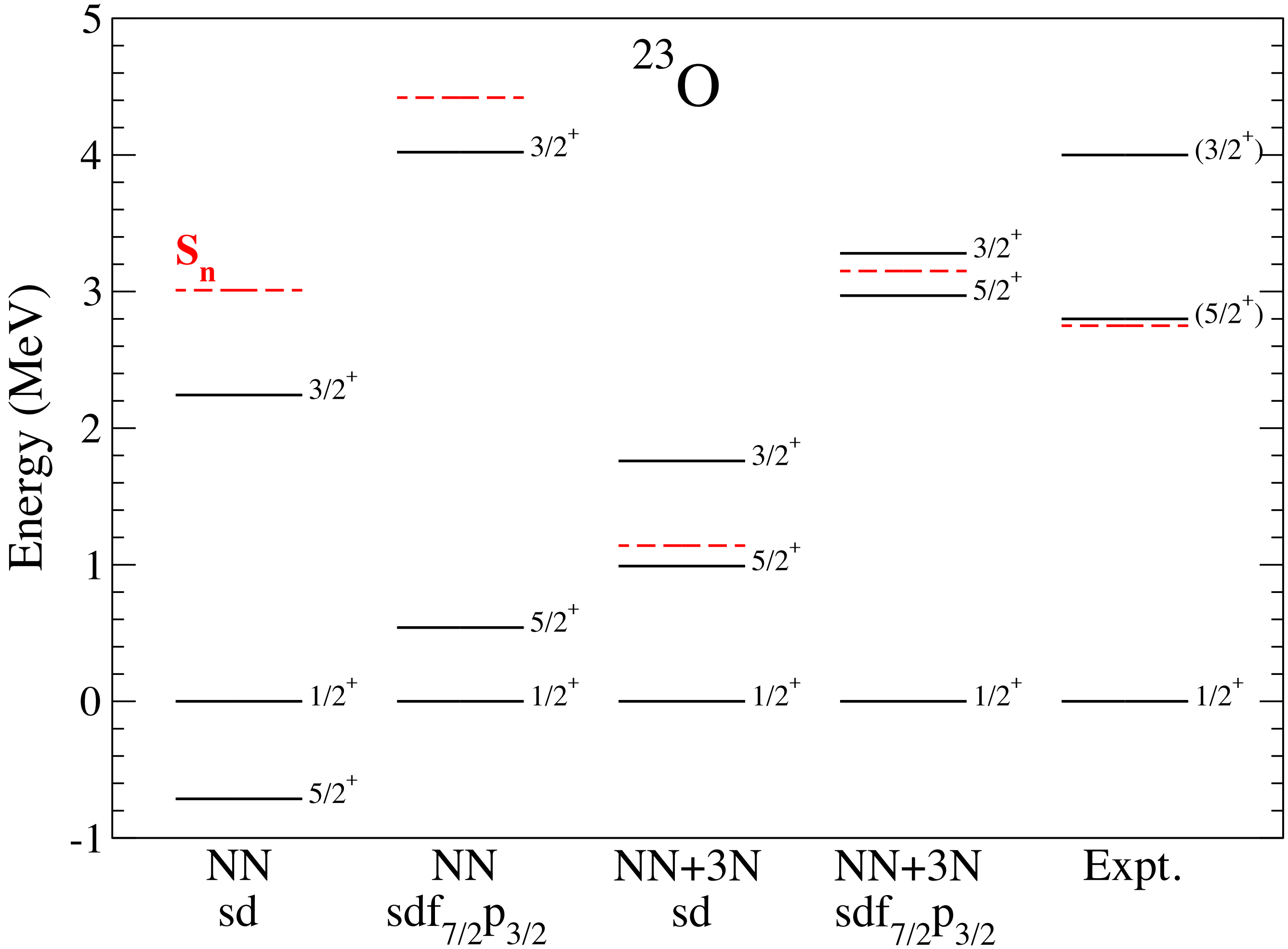
Unnaturally large couplings when 3N fit in ^{3}H(?) – results off the plot!

Lesson: 3N forces unavoidable part of theory – must investigate importance

Impact on Spectra: ^{23}O

Neutron-rich oxygen spectra with NN+3N

$5/2^+$, $3/2^+$ energies reflect $^{22,24}O$ shell closures



***sd*-shell NN only**

Wrong ground state

$5/2^+$ too low

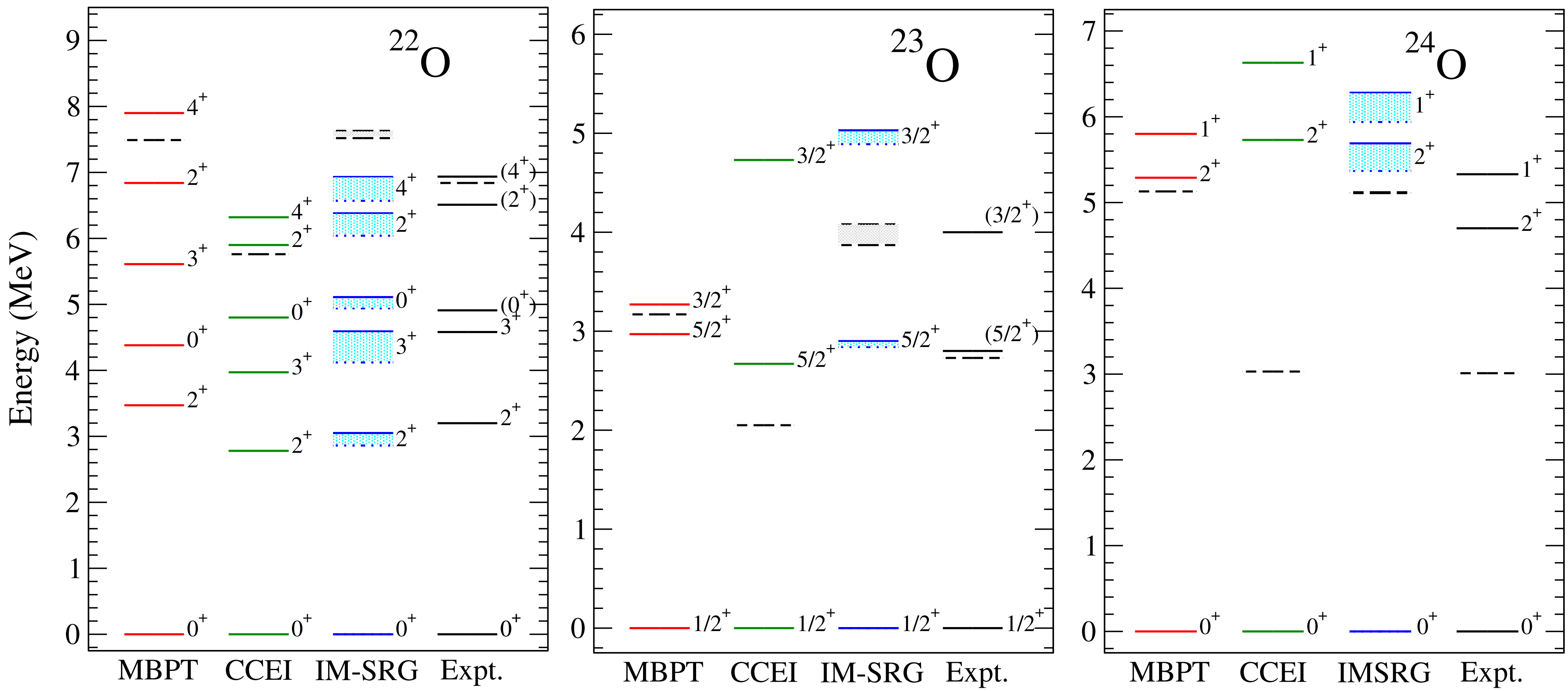
$3/2^+$ bound

NN+3N

Clear improvement in extended valence space

Comparison with MBPT/CCEI Oxygen Spectra

Oxygen spectra: Effective interactions from **Coupled-Cluster theory**

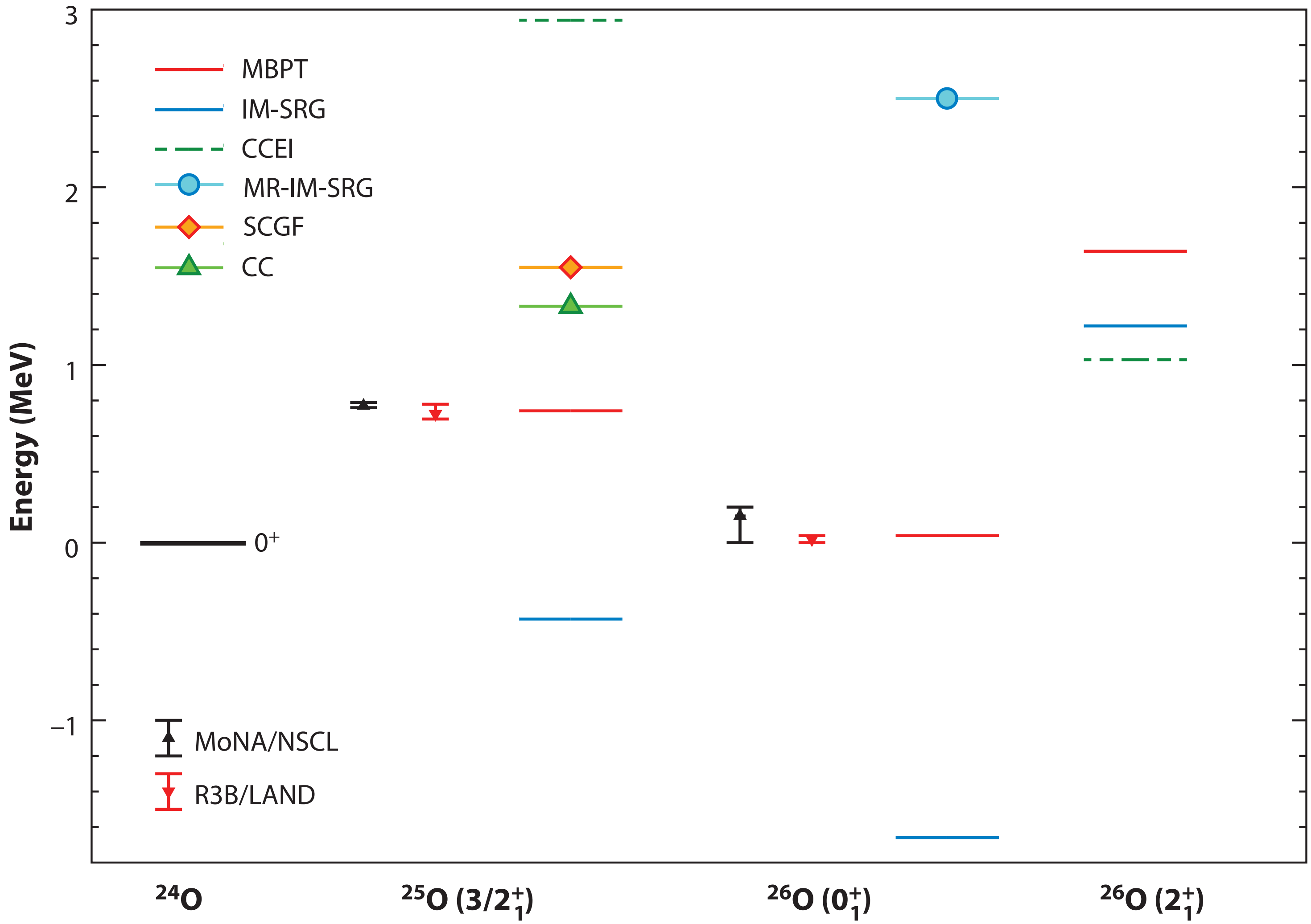


MBPT in extended valence space

IM-SRG/**CCEI** spectra agree within ~300 keV

Beyond the Oxygen Dripline

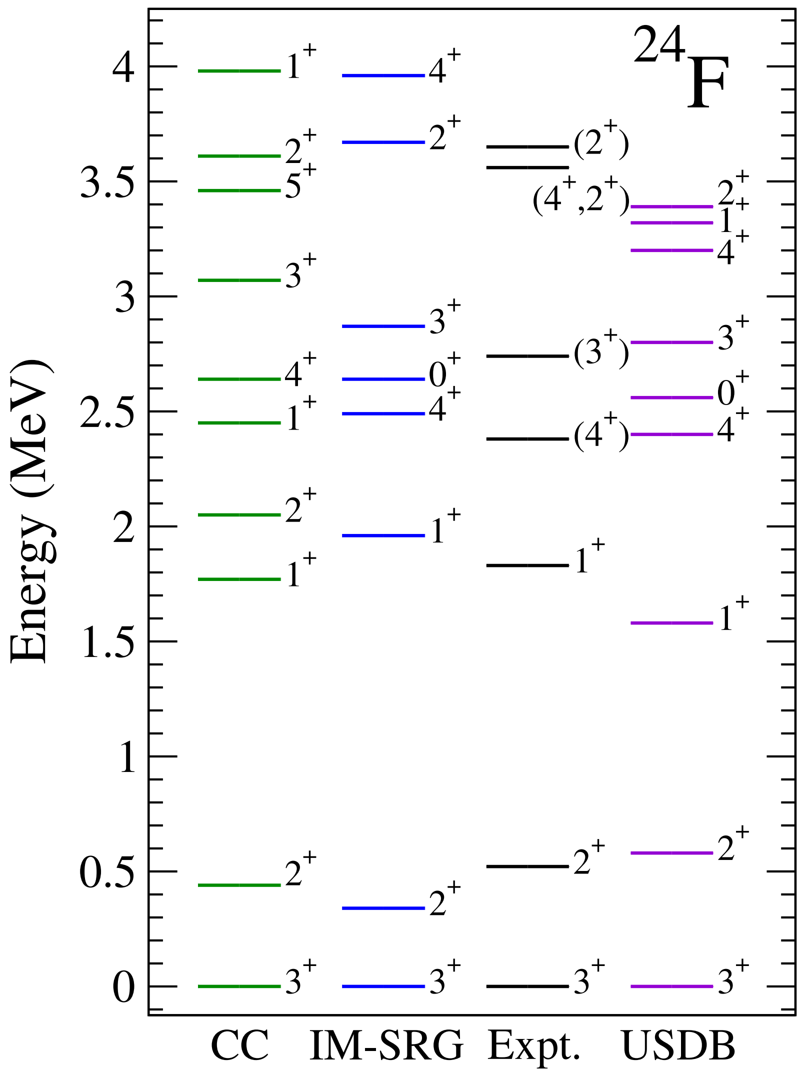
Physics beyond dripline highly sensitive to 3N and continuum effects



Prediction of low-lying 2^+ in ^{26}O (recently measured at RIKEN)

Experimental Connection: ^{24}F Spectrum

^{24}F spectrum: **IM-SRG** (*sd* shell), **full CC**, **USDB**



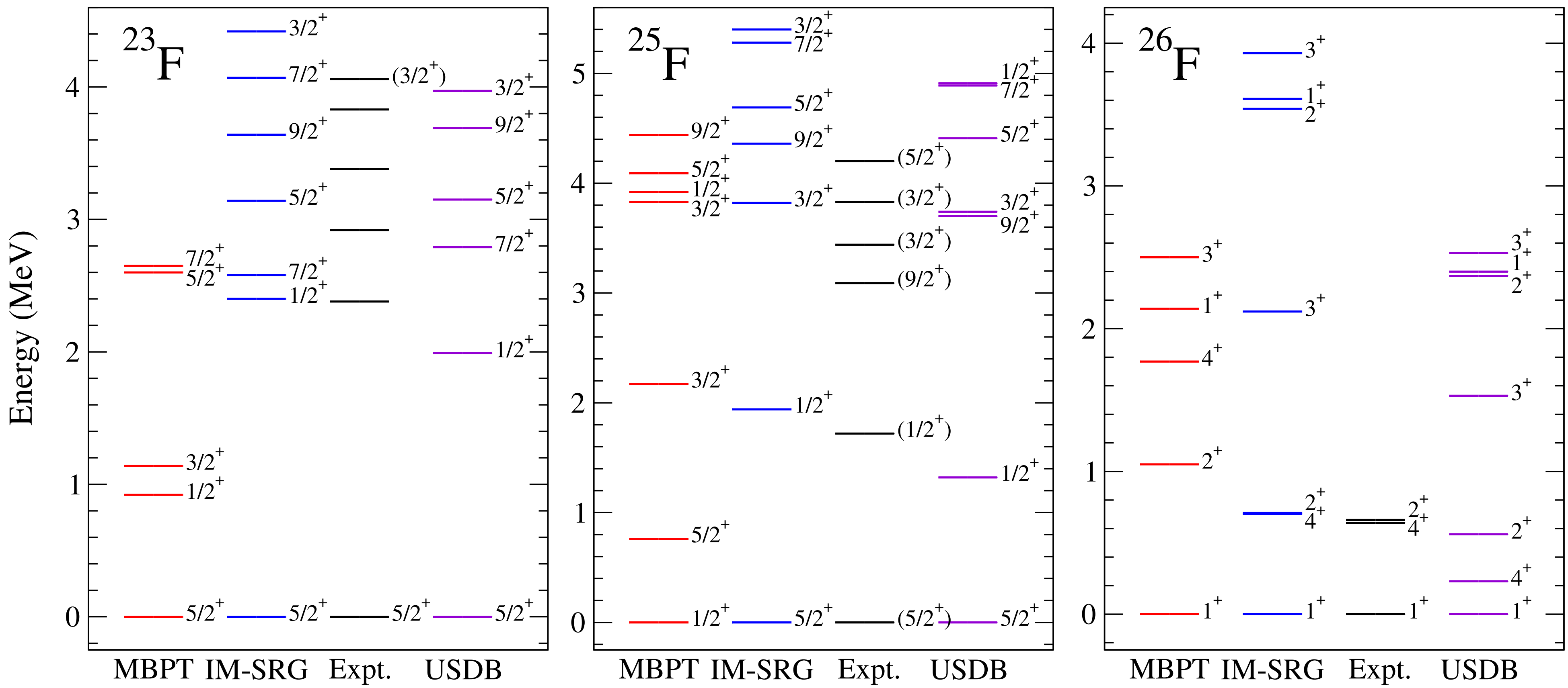
Ekström et al., PRL (2014)
Cáceres et al., arXiv:1501.01166
Hebeler, JDH, Menéndez, Schwenk, ARNPS (2015)

New measurements from GANIL

IM-SRG: comparable with phenomenology, good agreement with new data

Fully Open Shell: Neutron-Rich Fluorine Spectra

Fluorine spectroscopy: **MBPT** and **IM-SRG** (*sd* shell) from NN+3N forces



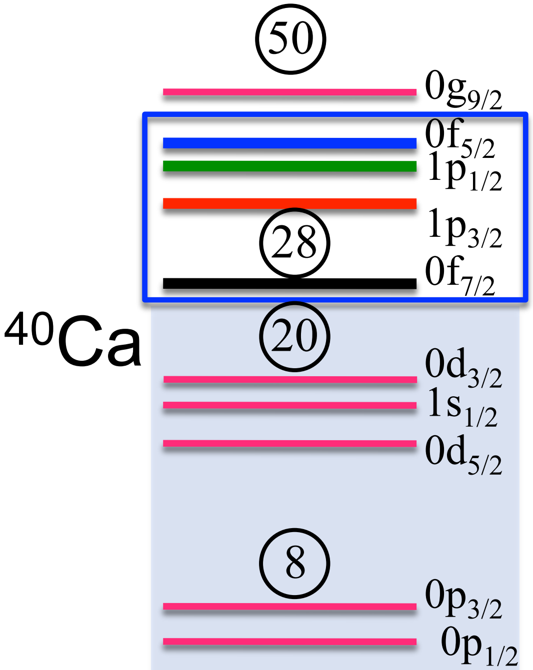
Bogner, Hergert, JDH, Schwenk, in prep.

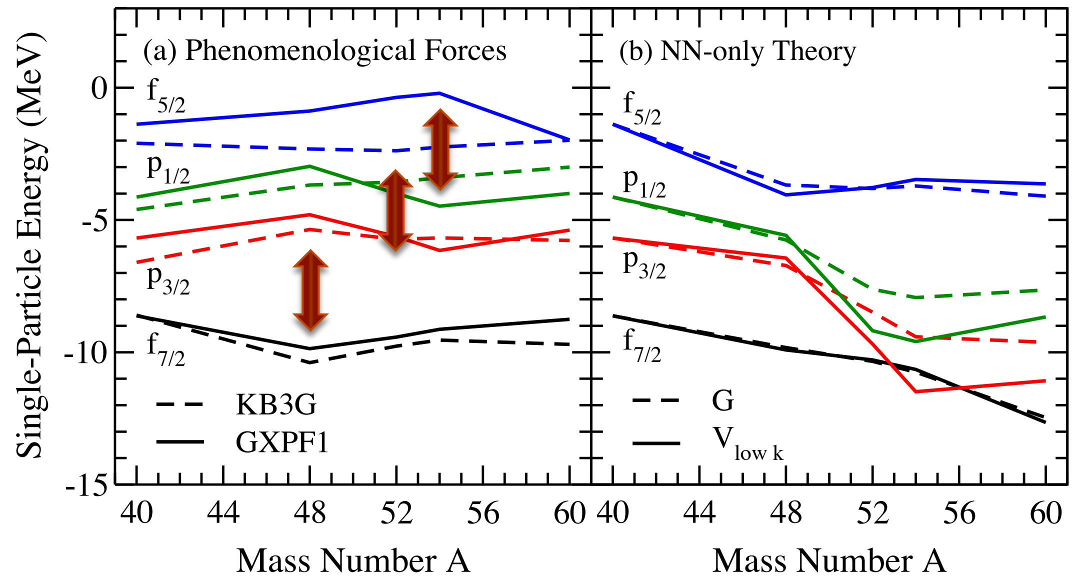
IM-SRG: **competitive with phenomenology**, good agreement with data

Preliminary results already for scalar operators: charge radii, E0 transitions

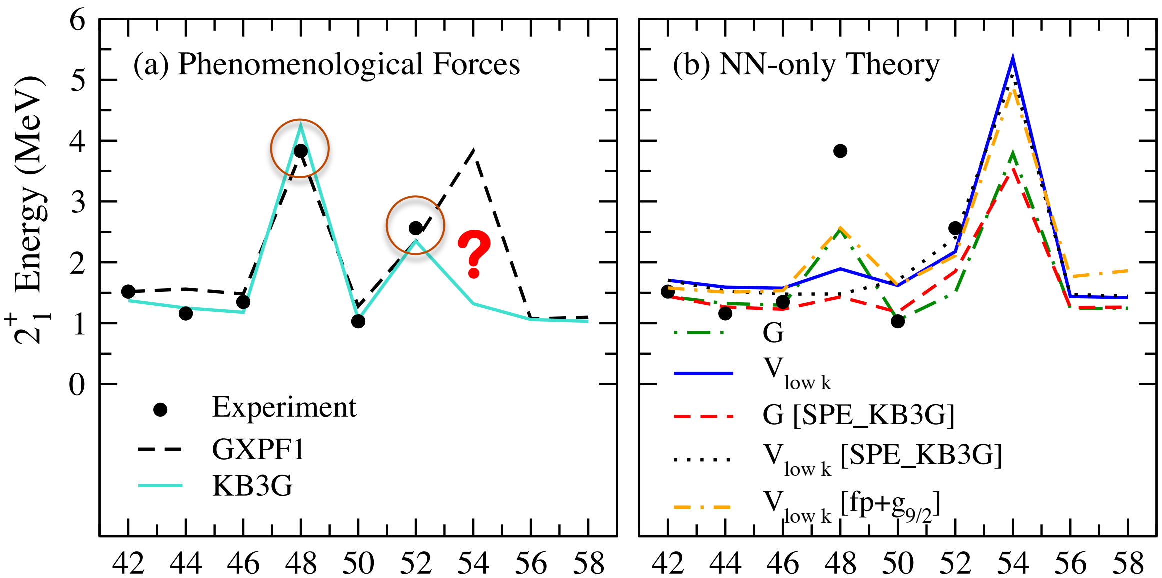
Upcoming: general operators M1, E2, GT, double-beta decay Stroberg et al.

Calcium Isotopes: Magic Numbers





GXPF1: Honma, Otsuka, Brown, Mizusaki (2004)
KB3G: Poves, Sanchez-Solano, Caurier, Nowacki (2001)



Phenomenological Forces

Large gap at ^{48}Ca

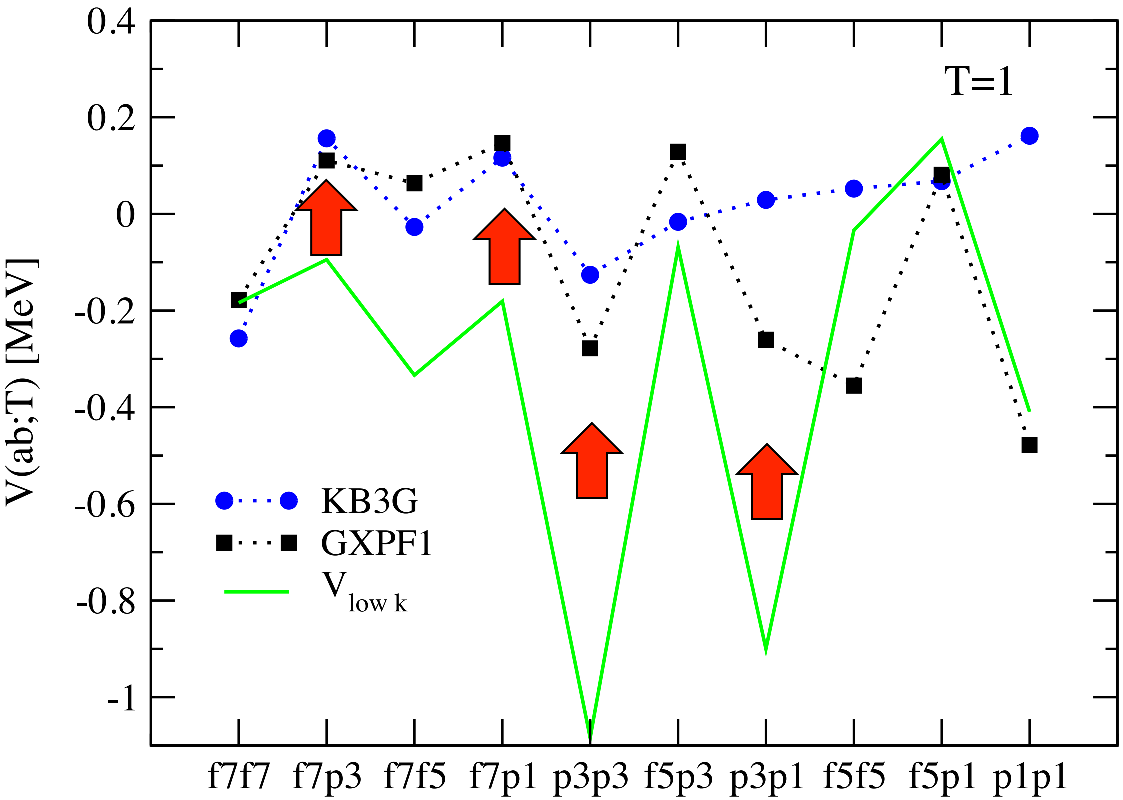
Discrepancy at $N=34$

Microscopic NN Theory

Small gap at ^{48}Ca

N=28: first standard magic number not reproduced in microscopic NN theories

Phenomenological vs. Microscopic



Compare monopoles from:

Microscopic **low-momentum** interactions

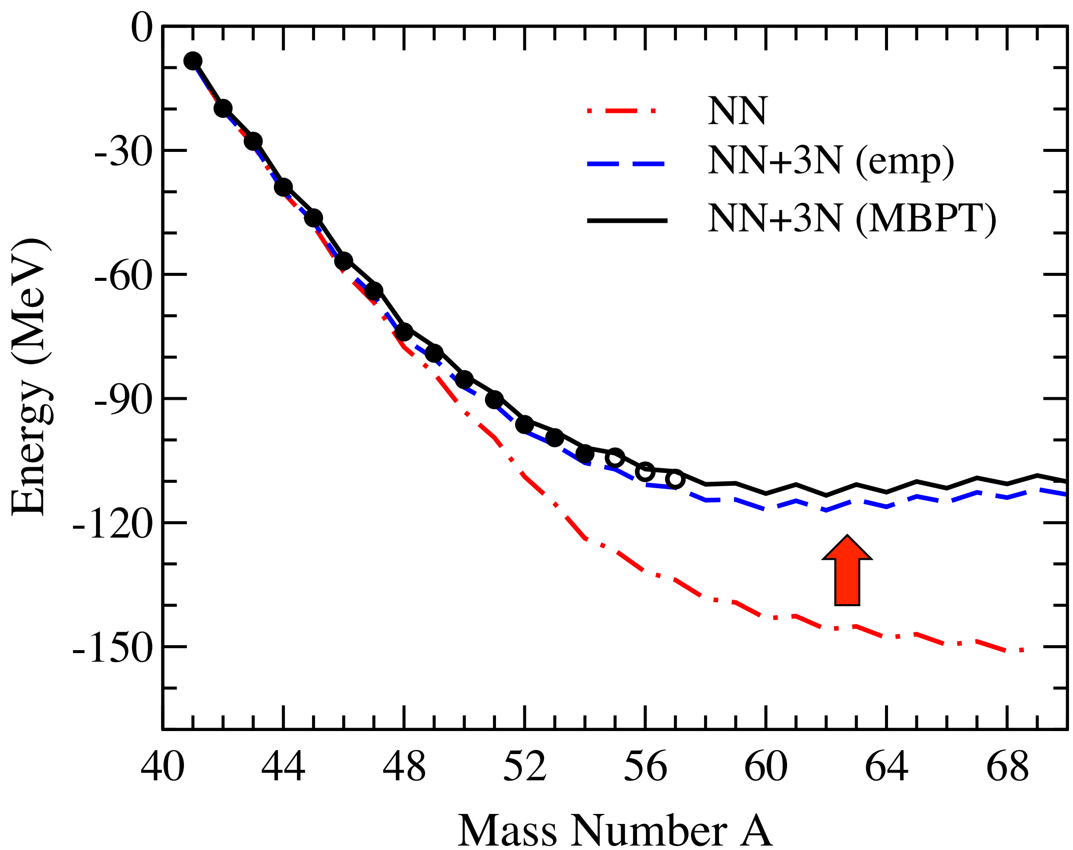
Phenomenological **KB3G, GXPF1** interactions

Shifts in **low-lying orbitals**:

- T=1 repulsive shift

Calcium Ground State Energies and Dripline

Signatures of shell evolution from ground-state energies?



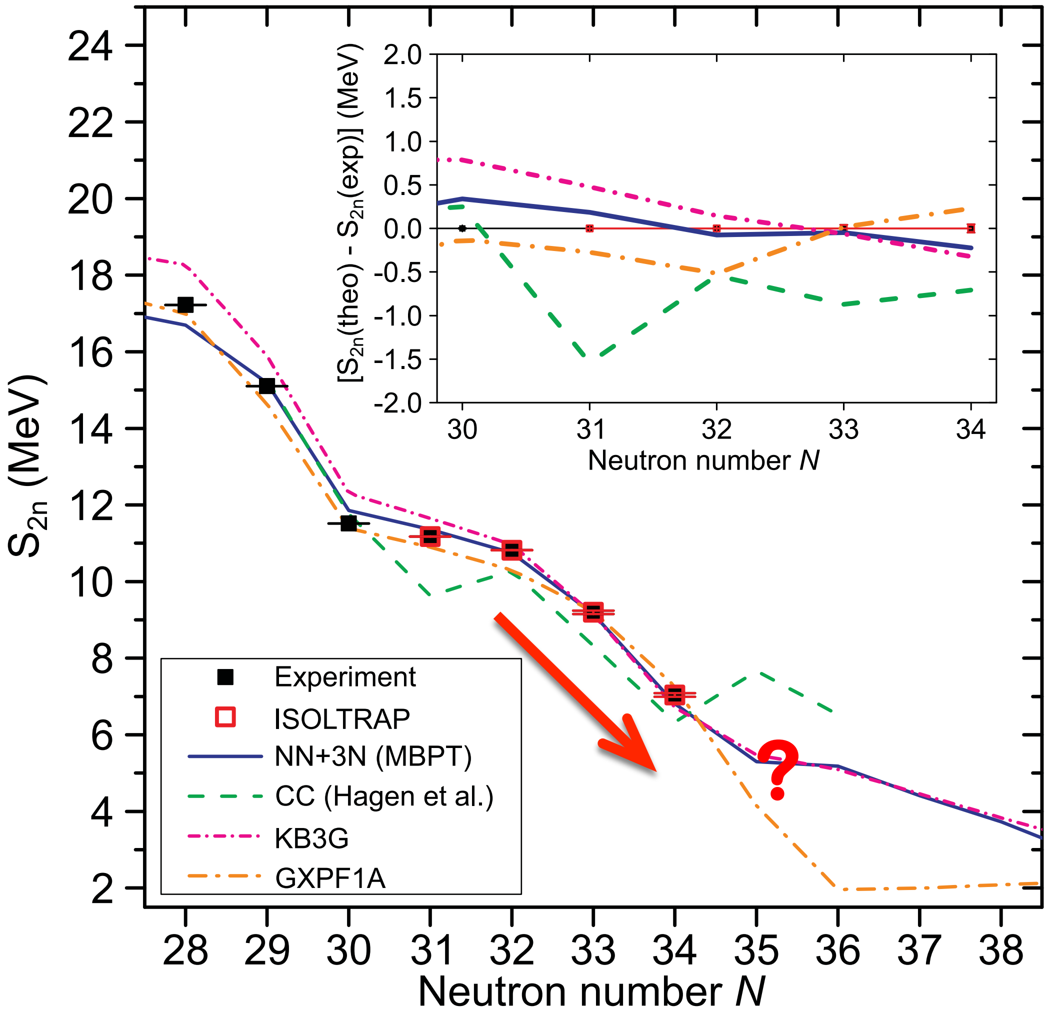
Holt, Otsuka, Schwenk, Suzuki, JPG (2012)

No clear dripline; flat behavior past ^{54}Ca – **Halos beyond ^{60}Ca?**

$S_{2n} = -\left[BE(N,Z) - BE(N-2,Z)\right]$ **sharp decrease indicates shell closure**

Experimental Connection: Mass of ^{54}Ca

New precision mass measurement of ^{53,54}Ca at **ISOLTRAP**: multi-reflection ToF



Wienholtz et al., Nature (2013)

TITAN Measurement

Flat trend from $^{50\text{-}52}$Ca

Mass ^{52}Ca 1.74 MeV from AME

ISOLTRAP *Measurement*

Sharp decrease past ^{52}Ca

Unambiguous closed-shell ^{52}Ca

Test predictions of various models

MBPT NN+3N

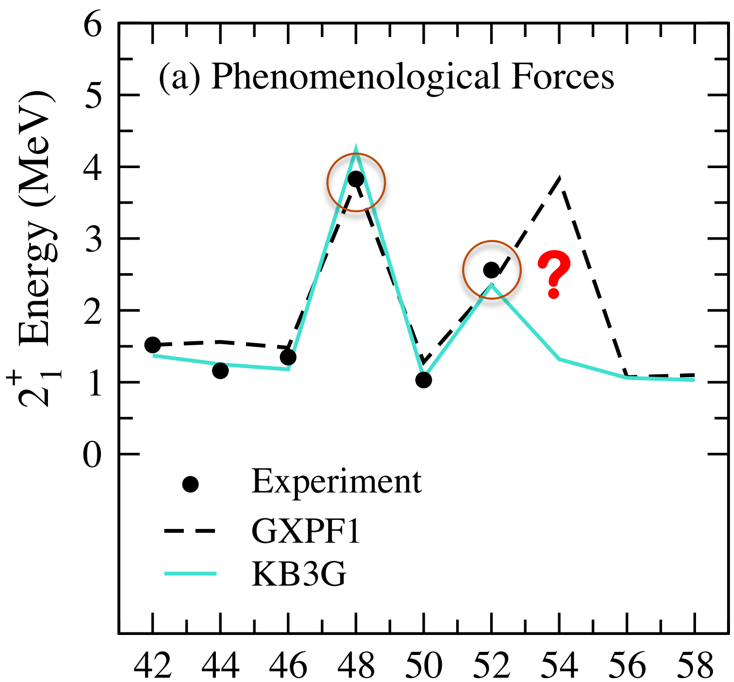
Excellent agreement with new data

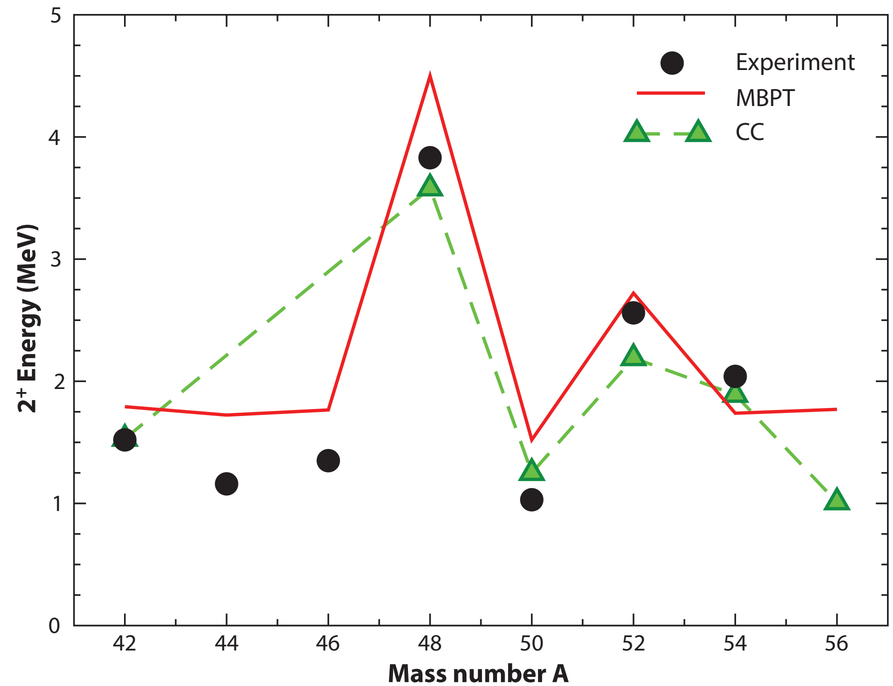
Reproduces closed-shell ^{48,52}Ca

Weak closed sell signature past ^{54}Ca

N=34 magic number in calcium?

Calcium Isotopes: Magic Numbers





GXPF1: Honma, Otsuka, Brown, Mizusaki (2004)
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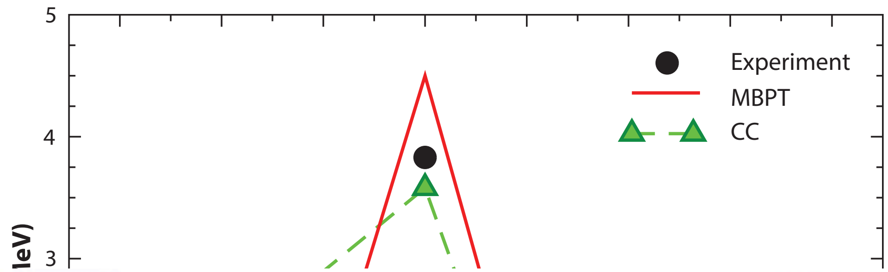
Phenomenological Models

Large gap at ^{48}Ca, discrepancy at $N=34$

Ab initio theories

Reproduce all new magic numbers, **consistent predictions**

Calcium Isotopes: Magic Numbers



PHYSICS

NEWS & VIEWS RESEARCH

Heavy calcium nuclei weigh in

The configurations of calcium nuclei make them good test cases for studies of nuclear properties. The measurement of the masses of two heavy calcium nuclei provides benchmarks for models of atomic nuclei. SEE LETTER P.346

ALEXANDRA GADE

quarks and gluons, which interact to form

LETTER

doi:10.1038/nature12226

Masses of exotic calcium isotopes pin down nuclear forces

F. Wienholtz[1], D. Beck[2], K. Blaum[3], Ch. Borgmann[3], M. Breitenfeldt[4], R. B. Cakirli[3,5], S. George[1], F. Herfurth[2], J. D. Holt[6,7], M. Kowalska[8], S. Kreim[3,8], D. Lunney[9], V. Manea[9], J. Menéndez[6,7], D. Neidherr[2], M. Rosenbusch[1], L. Schweikhard[1], A. Schwenk[7,6], J. Simonis[6,7], J. Stanja[10], R. N. Wolf[1] & K. Zuber[10]

LETTER

doi:10.1038/nature12522

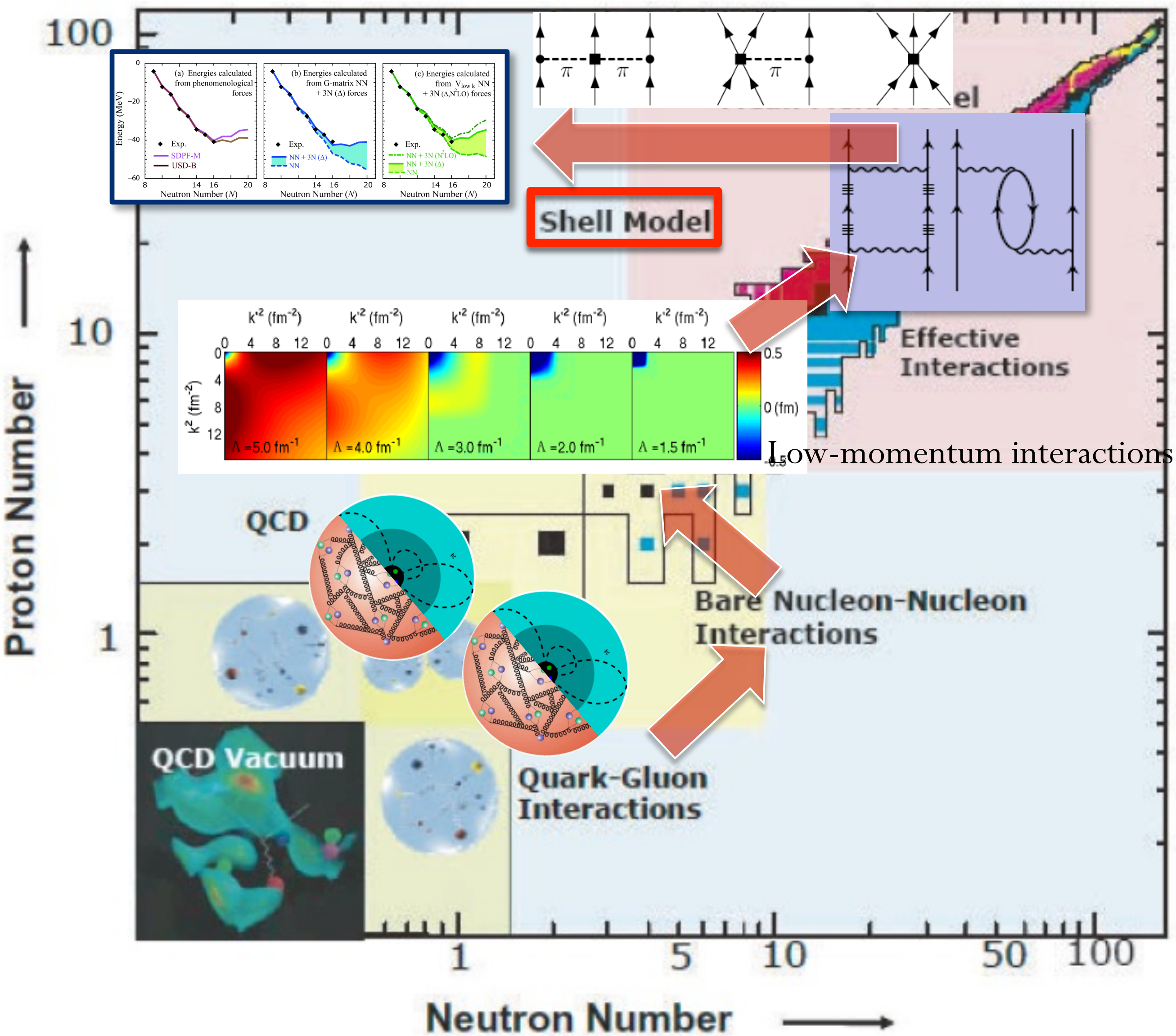
Evidence for a new nuclear 'magic number' from the level structure of ^{54}Ca

D. Steppenbeck[1], S. Takeuchi[2], N. Aoi[3], P. Doornenbal[2], M. Matsushita[1], H. Wang[2], H. Baba[2], N. Fukuda[2], S. Go[1], M. Honma[4], J. Lee[2], K. Matsui[5], S. Michimasa[1], T. Motobayashi[2], D. Nishimura[6], T. Otsuka[1,5], H. Sakurai[2,5], Y. Shiga[7], P.-A. Söderström[2], T. Sumikama[8], H. Suzuki[2], R. Taniuchi[5], Y. Utsuno[9], J. J. Valiente-Dobón[10] & K. Yoneda[2]

t predictions

The Challenge of Microscopic Nuclear Theory

To understand the properties of complex nuclei from elementary interactions



Three-Nucleon Forces

Clear path from symmetries of QCD to shell model

Ideas of:

Effective field theories

Renormalization group

Advances in many-body

Advances in computing

All essential for this progress

Still much to do!!

How will we approach this problem:

QCD → NN (3N) forces → Renormalize → Solve many-body problem → Predictions

New Directions and Outlook

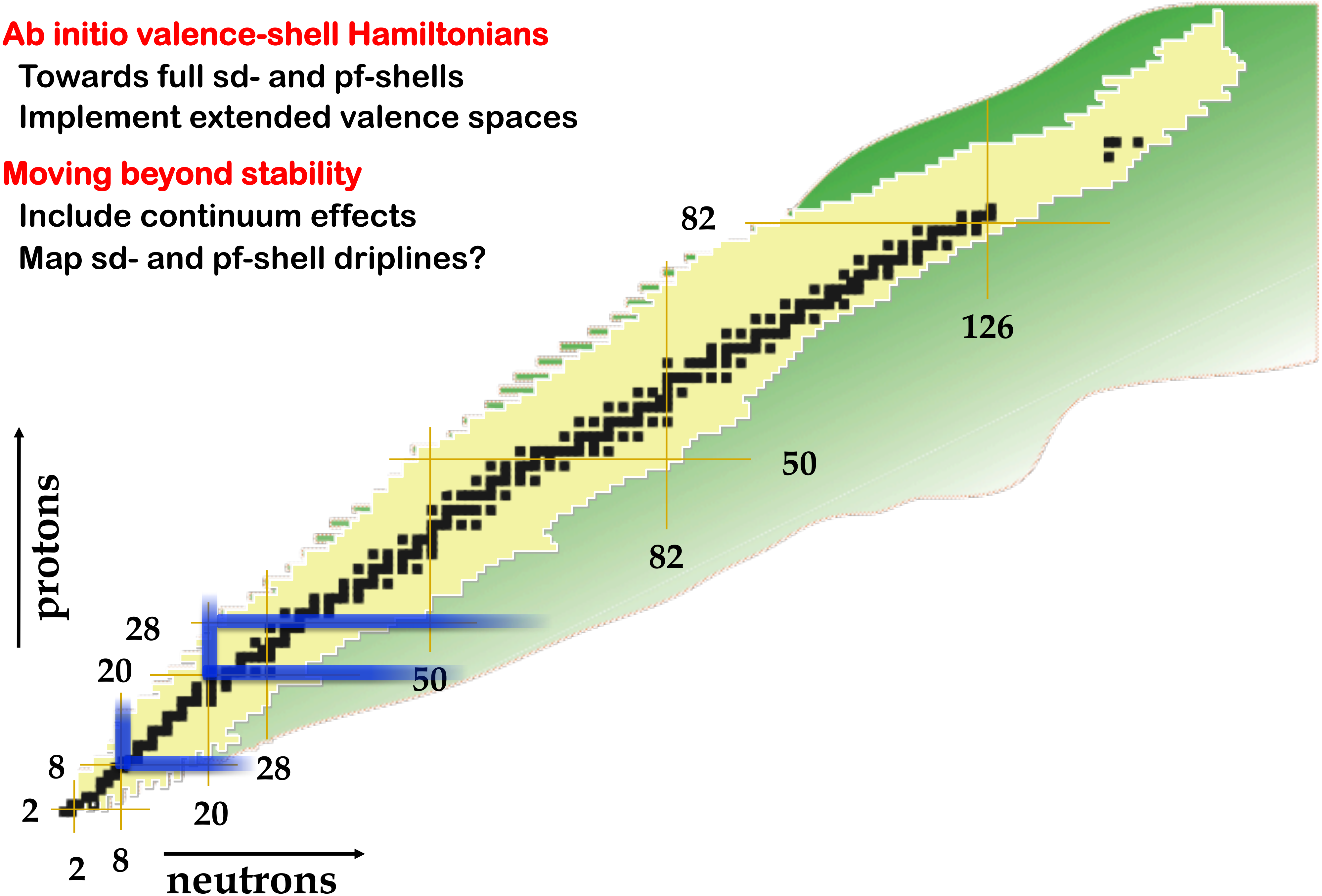
Heavier semi-magic chains: MBPT as guide

Ab initio valence-shell Hamiltonians

- Towards full sd- and pf-shells
- Implement extended valence spaces

Moving beyond stability

- Include continuum effects
- Map sd- and pf-shell driplines?



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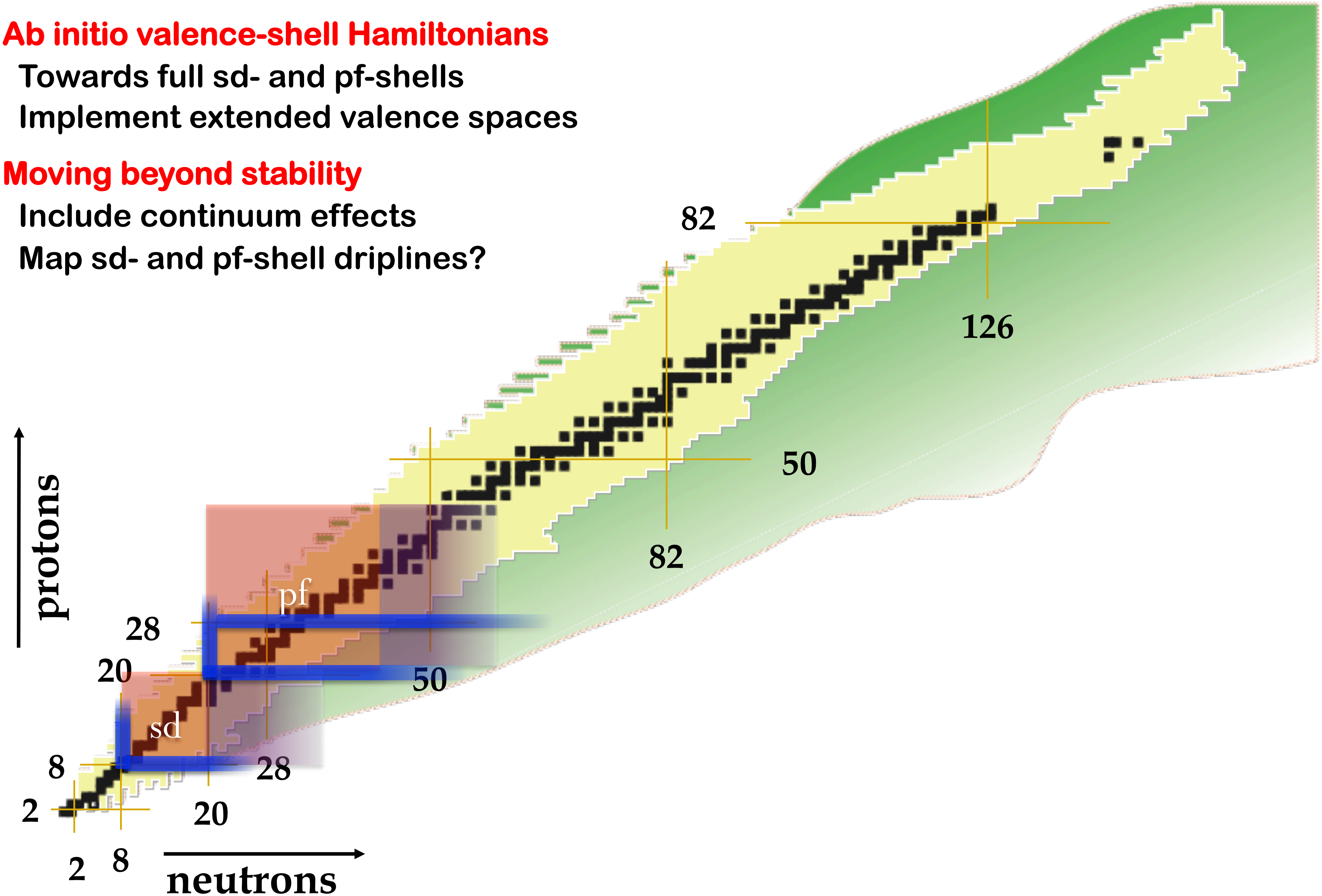

New Directions and Outlook

Heavier semi-magic chains: MBPT as guide

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Towards full sd- and pf-shells

Implement extended valence spaces

Moving beyond stability

Include continuum effects

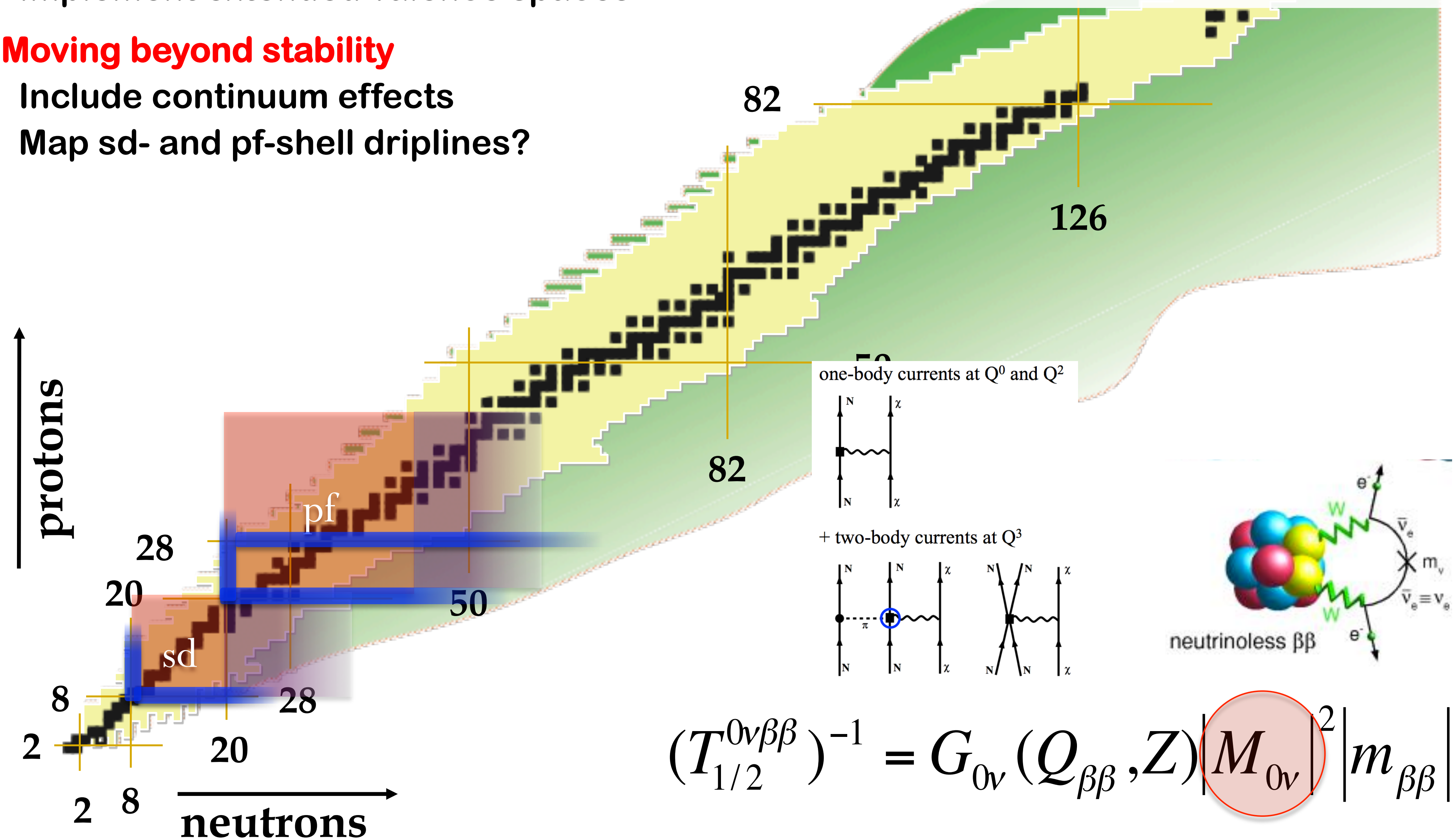
Map sd- and pf-shell driplines?

Fundamental symmetries

Effective electroweak operators

ab initio calculation of $0\nu\beta\beta$ decay

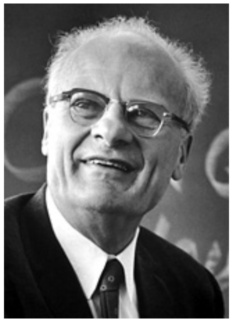
WIMP-nucleus scattering



$$(T_{1/2}^{0\nu\beta\beta})^{-1} = G_{0\nu}(Q_{\beta\beta}, Z)\left|M_{0\nu}\right|^2 \left|m_{\beta\beta}\right|^2$$

Final Thought

"Very soft (NN) potentials must be excluded because they do not give saturation; they give too much binding and too high density."
 - ***H. Bethe***



How might you respond?

Final Thought

"Very soft (NN) potentials must be excluded because they do not give saturation; they give too much binding and too high density."
 - ***H. Bethe***

How might you respond?

Further Reading

Lepage, nucl-th/9706029 (1997)

Epelbaum, Hammer, Meißner, Rev. Mod. Phys. (2009)

Machleidt, Entem, Phys. Rep. (2011)

Bogner, Furnstahl, Schwenk, Prog. Part. Nucl. Phys. (2010)

Hebeler, Holt, Menendez, Schwenk, Ann. Rev. Nucl. Part. Sci. (2015)

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Scott Bogner, Angelo Calci, Thomas Duguet, Dick Furnstahl, Alex Gezerlis, Gaute Hagen, Kai Hebeler, Heiko Hergert, Herman Krebs, Javier Menendez, Petr Navratil, Achim Schwenk, Johannes Simonis, Ragnar Stroberg